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On the Equivalent Sources and Geometric Factor Calculation for a Circular Detection Case

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Abstract In all absolute measurements of the intensity of the radioactive materials and calibration of the detectors, it is essential the knowledge of the geometric efficiency. This work describes how to obtain the sources with different geometries and equal geometric efficiency (equivalent sources for geometric factor), corresponding to a linear, circumferential and circular homogeneous sources parallel to a circular detector. It is estimated the geometric factor of them by the Monte Carlo method. The results are compared with the published in the literature, thus confirming the validity of this method.

Keywords: geometric factor, Monte Carlo method, equivalent source.

1. INTRODUCTION

In dosimetry and nuclear spectrometry in general, the calibration of the detectors with cylindrical geometry, like others, require multi-isotopic radiation sources in a broad photon energy range. In some applications of nuclear spectrometry, for example in the gamma geometry data acquisition system, the range of energy for environmental studies and orphan or unknown radioactive waste sources is large, between 1keV and 10MeV, requiring for optimal calibration a relatively large number of primary standards and as is well known requires considerable investment. Because this procedure is not practical as in some cases requires a standard that is representative of the physical and nuclear properties of the materials under study. That is why several other techniques

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have been developed to determine the geometric efficiency; this parameter is essential in all absolute measurements of the radiation intensity emitted by radioactive materials. Among the techniques available in the literature, we mention the semi-empirical methods and Monte Carlo simulations in the calibration of detectors [15, 11, 1]. Often the Monte Carlo method is used [12] to calculate the geometric efficiency of detection systems with circular windows and sources with specific geometries as point, linear, circumferential, circular or others [2, 4, 5, 6, 9]. Both analytical and numerical analysis can be complicated significantly by increasing the number of dimensions of the source under study or considering irregular shapes. So the availability of simulate the source under study through another one that allows to perform simple calculations, as regards the geometric efficiency, offers a convenient method for the technicians of the dosimetric area and nuclear spectrometry. In a previous work Viloria et al. [8], the term equivalent sources was introduced to refer to sources with different geometries and the same geometric efficiency for detectors with established geometries; in this study we refer to them as Equivalent Sources for Geometric Factor (ESGF) conducting a more detailed analysis in order to develop a new Numerical Algorithm Method (NAM), through Monte Carlo method.

2. AZIMUTH INVARIANCE OF THE GEOMETRIC FACTOR AND THE ESGF.

Based on the invariance of the geometric factor of the point sources along a concentric circumference parallel to a circular detector, it is possible to simulate a circummferencial source of radius or a circumferential arc source of the same radius, as a point source in the same plane at a distance from the symmetry axis of the detector, equal to.

Each specific source has an infinite number of ESGF, of different length, shapes and dimensions. The authors believe that the simplest way to obtain an ESGF, parallel to a circualr detector, is the one shown below, where the ESGF is inscribed in one circumference of radius equal to the distance from the symmetry axis of the detector to the farthest point from the source under study, to the symmetry axis of the detector (e.g. circular sources intersected by the axis of symmetry or the cirucmferencial sources of radii equal to the displacement of it from the symmetry axis of the detector) or between two concentric circumferences of radii equal to the distance to the nearest and the farest point from the source under study, to the symmetry axis of the detector (e.g. linear and circumferential sources of radii different to the displacement from the symmetry axis of the detector).

The foregoing makes it possible to simulate extended sources (one-, two- and three-dimensional sources) by sources of fewer dimensions [8].

3. NAM TO DETERMINE THE GEOMETRIC FACTOR BY THE USE OF THE ESGF.

Unlike the procedure by Monte Carlo method used so far to calculate the geometric factor for detector-source systems, in which sampling is carried out by the entire source [12], in this paper it is propose to carry out the sampling along the ESGF.

To calculate the geometric factor by the Monte Carlo method of hit and miss, the expression used for us in the present work was

$$\varepsilon_G = \frac{\sum_{n=1}^{n_{max}} p_n A_n}{\sum_{n=1}^{n_{max}} p_n \left(A_n + D_n\right)} \text{ QUOTE} \tag{1}$$

where A_n is the number of particles emitted toward the detector, while D_n corresponds to the number of particles with different orientation, represents the weight and n_{max} is the number of particles simulating the ESGF, all them corresponding to the n-th point source.

Next in the Figure 1, is shown the flowchart of the proposed computer algorihtm.

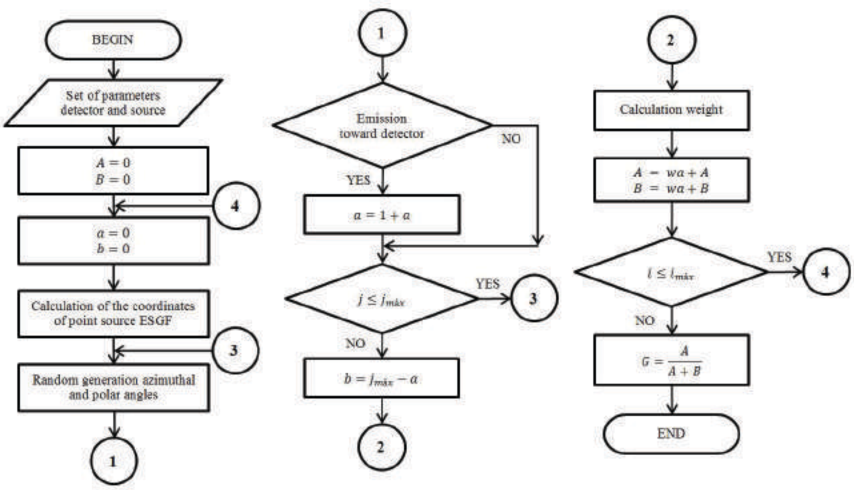


Figure 1: Flowchart of the algorithm.

3.1 Homogeneous linear sources parallel to a circular detector

For a linear source under study, should be chosen a variable (u_i) describing the distribution of the point sources that make up the linear source. When all position vectors $r_i = r_i(u_i)$ are aligned radially it is possible to obtain a linear radially oriented ESGF.

For an homogeneous linear source parallel to a circular detector, which midpoint is displaced from the axis of symmetry of the detector to a longitudinal and lateral distance at, the general equation of the norm of the position vectorsof the particles that simulate the linear ESGF radially oriented corresponding to the source under study will be as follow:

$$r_i = \sqrt{a_T^2 + \left\{ n\left(a_L - \frac{L}{2} \right) + \frac{i}{N}\left[(1-n)a_L + (1+n)\frac{L}{2} \right] \right\}^2} \qquad (2)$$

Where i varies from 0 to N; N is the number of particles which simulate the linear source. The parameter takes values 1 or 0. For our analysis approach is convenient to consider two cases. In thecase number one, all point sources of the linear source under study are at different distances from the axis of symmetry (Figure 2a), and in the second case, some or all of the points sources that simulate the linear source have a "mirror" point source (Figure 2b). In the case number one, $n=1$ and the multiplicative weightequals one ($p_i = 1$), but in the second case, and sampling is not performed along the entire line source, but from the point of intersection with the perpendicular axis and the far end point. For those point sources with "mirror" point sources, the weigth equals two ($p_i = 2$) and for the rest equals one ($p_i = 1$).

In graphic1a, is displayed the distribution of the linear radially oriented ESGF, corresponding to a homogeneous linear source which orientation varies respect to the radii of the concentric circumferences, as it is shown in the upper left box.

3.2 Circumferential homogeneous sources parallel to a circular detector

The geometric factor for the circumferential sources have been studied by several authors [3, 8]. In this case the norm of the position vectors of each point source, belonging to the linear radially oriented ESGF, corresponding to a circumferential source displaced from the axis of symmetry of the detector is obtained from the following equation

$$r_k = \sqrt{r_S^2 + d^2 - 2r_S d\cos\alpha_k} \qquad (3)$$

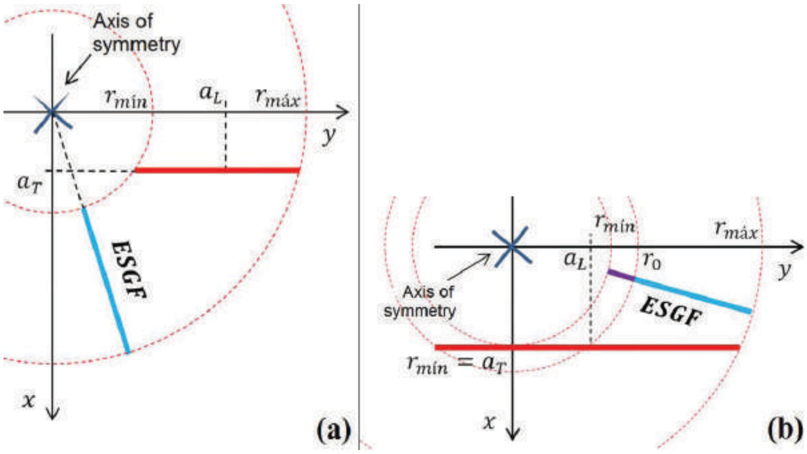
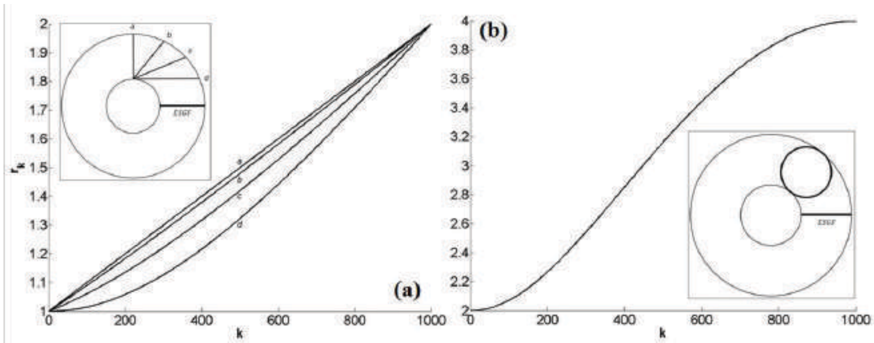


Figure 2: (a) Linear radially oriented ESGF for the case one of a linear source under study of arbitrary orientation parallel to the circular detector. (b) Linear radially oriented ESGF for the case two of a linear source under study of arbitrary orientation parallel to the circular detector.



Graphic 1: (a) Variation of the distribution of the point sources of the linear radially oriented ESGF, corresponding to a homogeneous linear source as shown in the upper left box. The index $\boldsymbol{k}$ corresponds to the $\boldsymbol{k}$-th ESGF point source, and $\boldsymbol{r_k}$ the corresponding position vector. (b) Distribution of the point sources of a radially oriented linear ESGF corresponding to a circumferential source as shown in the lower right box.

where the subscript represents the k-th point source, r_S the radius of the circumferential source and α_k the angle between the displacement vector of the center of the circumferential source and the radius of the circumferential source with end in the k-th point source $\left(\alpha_k = k\pi / N\right)$ (Figure 3). The ESGF for the circumferential source will be inscribed in one or between two concentric circumferences of radii equal to the distances from the nearest and the farthest point of the source to symmetry axis of the detector (Figure 4).

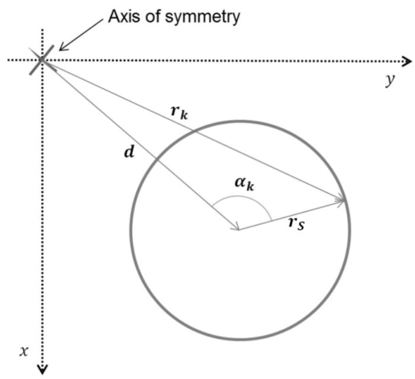


Figure 3: Circumferential source under study parallel to the circular detector.

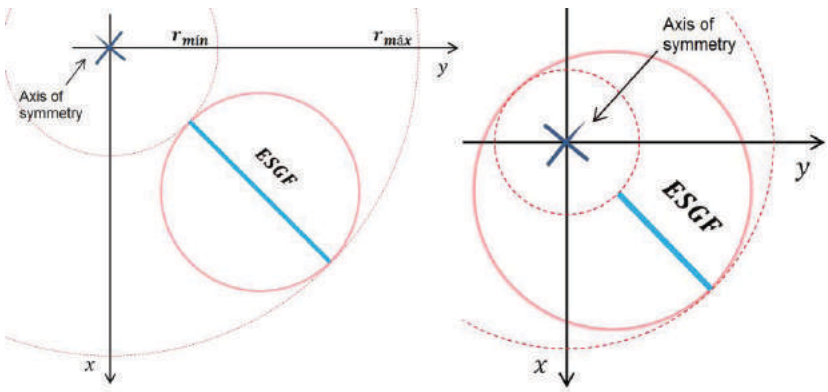
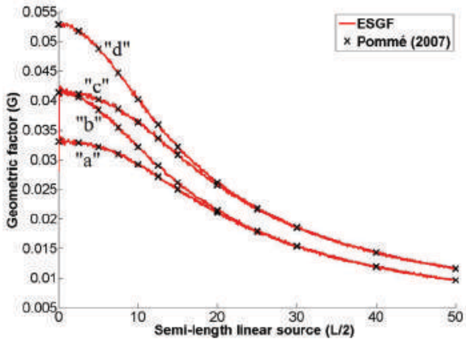


Figure 4: Length ESGF corresponding to relative position of the circumferential source under study parallel to the circular detector.

In the calculation of the geometric factor of a circumferential source, the multiplicative weight equals 2 ($p_k = 2$) , except for $\alpha_k = 0$ and $\alpha_k = \pi$, for which it equals one ($p_k = 1$) .

In graphic 2, is displayed the distribution of the point sources of the linear radially oriented ESGF corresponding to a homogeneous circumferential source, inscribed between two concentric circumferences, respect to the detector axis of symmetry, as it is shown in the lower right box.



Graphic 2: Geometric efficiency for a circular detector of radius $\boldsymbol{r_D}$ **= 5**, and a homogeneous linear sources (cross) at an axial distance from the detector of $\boldsymbol{h} = 10$, displaced from the symmetry axis (a) $\boldsymbol{a_T} = 0$, $\boldsymbol{a_L} = 0$, (b) $\boldsymbol{a_T} = 0$, $\boldsymbol{a_L} =$ **5**, (c) $\boldsymbol{a_T} = 5$, $\boldsymbol{a_L} = 0$, (d) $\boldsymbol{a_T} = 5$, $\boldsymbol{a_L} = 5$, as a function of its semi-length, and the corresponding non-homogeneous linear ESGF of radial orientation (solid line).

3.2 Circular homogeneous sources parallel to a circular detector.

In the case of a circular source parallel to a circular detector, the source is represented as circumferences and/or circumferential arcs, concentric to the axis of symmetry of the detector as shown. Each circumference or circumferential arc intersecting the circular source is simulated as a point source, all this points aligned radially conform the linear radially oriented ESGF. The weight $\left(p_k\right)$ will be proportional to the radius of the k -th concentric circumferences or to the -th circumferential arcs r_k.

$$p_k = \frac{\varphi_k}{2\pi} r_k \tag{4}$$

where, φ_k is the angle swept by the radius when moves along the k -th concentric circumferential arc intersecting the circular source, if $r_k \le d$ the angle will be $\varphi_k = 2\pi$ (Figure 5). The position of the radionuclides k-this determined by the next expression

$$r_k = \sqrt{\mathrm{d}_x^2 + \left(\mathrm{d}_y - \frac{k}{N} r_s\right)^2} \tag{5}$$

where d_x and d_y are the center position of circular source respect the axis symmetric of detector, r_S is the radius of the source and N is the number of radionuclides that constitute the ESGF.

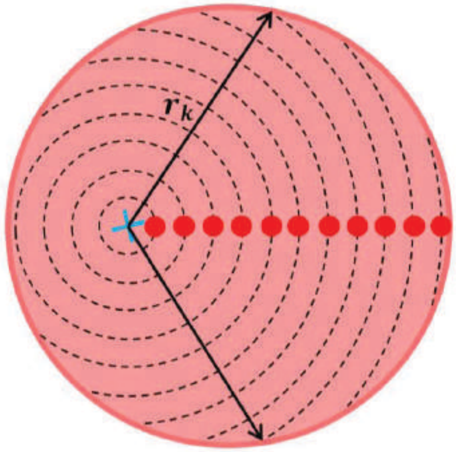


Figure 5: Concentric circumferences and arcs that divide the circular source parallel to the circular detector. The points represent the ESGF.

4. THE LENGTH OF THE LINEAR RADIALLY ORIENTED ESGF

The length of the linear radially oriented ESGF obtained by the above procedure and corresponding to a linear, circumferential or circular source under study (parallel to the circular detector), depends on the geometric characteristics of the sources under study and on the relative position to the symmetry axis of the detector.

4.1 Linear source under study

The maximum length of the linear radially oriented ESGF, can be obtained if the linear sources under study correspond to case one (see Figure 2a), and it can be calculated as follows:

$$L_{ES,max} = \sqrt{\left(a_L + \frac{L}{2}\right)^2 + a_T^2} - \sqrt{\left(a_L - \frac{L}{2}\right)^2 + a_T^2} \qquad (6)$$

If the linear sources under study correspond to case two (see Figure 2b), the length of the ESGF corresponding to this case will be of less dimension, and can be calculated by the following equation:

$$L_{ES,min} = \sqrt{\left(a_L + \frac{L}{2}\right)^2 + a_T^2} - a_T \qquad (7)$$

4.2 Circumferential source under study

The length of the linear radially oriented ESGF for a circumferential source under study can be represented by the following equation:

$$L_{ES} = r_F + d - |r_F - d| \tag{8}$$

The maximum length is obtained when $d \geq r_F$,

$$L_{ES,max} = 2r_F \tag{9}$$

From equation (8) in order to obtain the minimum length we proceeded as follows

$$\lim_{d \to 0} L_{ES} = 0 \tag{10}$$

the ESGF tends to a point source belonging to the circumference, that is what was expected; the ESGF for a coaxial circumference is a point source belonging to it.

4.3 Circular source under study

For the calculation of the length of a linear radially oriented ESGF corresponding to a circular source, it is used the following equation

$$L_{ES} = r_F + d - j|r_F - d| \tag{11}$$

Here are considered two cases, $d < r_F$ and $d \geq r_F$; for the first case $j = 0$,

$$L_{ES} = r_F + d \tag{12}$$

In this case the minimum length of the ESGF, for an specific circular source under study, is obtained when $d = 0$,

$$L_{ES,min} = r_F \tag{13}$$

In the second case $(j=1)$, $L_{ES} = r_F + d - |r_F - d|$. In this case for any value of d the length of the ESGF, is the maximum possible

$$L_{ES,max} = 2r_F \tag{14}$$

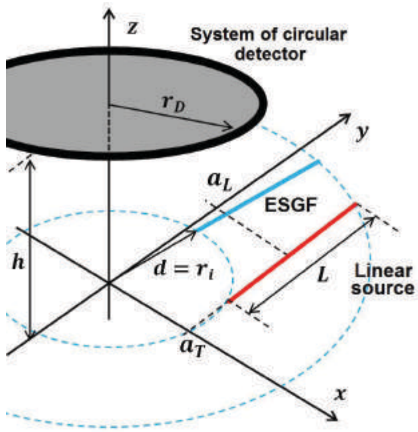


Figure 6: Circular detector – linear source parallel system, of length ***L***, position respect symmetric axes of detector $\boldsymbol{a}_L$ and $\boldsymbol{a}_T$, and $\boldsymbol{r}_D$ detector's radius, with separation between planes of h, and displacement of the source, respect to the detector's symmetry axis of ***d***.

5. RESULTS

Linear source. In order to validate the above, was used as a reference the equation published by [7]. In graphic 2 are shown the results obtained by Pomme, et al., for the values of the geometric factor of homogeneous linear sources (cross) parallel to circular detector of radius $r_D = 5$, and at a distance $h = 5$ from the detector, versus the semi-length of the linear sources which mid points are displaced from the axis of symmetry, (a) $a_T = 0$, (b) $a_T = 0$, $a_L = 5$, (c) $a_T = 5$, $a_L = 0$, (d) $a_T = 5$, $a_L = 5$; and the inhomogeneous linear ESGF (solid line). It is note worthy that in the simulation of the geometric factor is taken into consideration that by varying the length of the linear sources under study, they can change from a linear source of case 1, to a linear source of case 2, varying accordingly the weights.

The number of stories for obtaining values table 1 was $N = 10^6$, with a mean machine time $\overline{t_S} = 3,77$ seg.

Circumferential source. The validation of the equivalence between the geometric factor of a homogenous circumferential source of radio r_S which center is displaced at a distance d from the axis of symmetry of the detector of radius r_D, and located in a parallel planeto the detector at a distance h from it, and an inhomogeneous linear ESGF radially oriented, was conducted with reference to the equation of Conway's work [3].

Table 1: Geometric factor values for circular detector-linear source system, obtained by Pommé S. et al. [7] (G_p) and for the ESGF method (G_s), corresponding to the curve (a) of graphic 3. Confidence Interval (***CI***), absolute error (ε_A), relative erro (ε) and relative standard deviation (ε_s).

$L/2$	G_S	G_P	CI	ε_A	$\varepsilon_{\%}$	ε_S
0	0.0200	0.0331	0.0000	0.0131	39,5295	0.4383
2,5	0.0330	0.0329	0.0003	0.0002	0.5159	0.0131
5	0.0321	0.0321	0.0003	0.0000	0.0147	0.0107
7,5	0.0308	0.0309	0.0004	0.0001	0.3306	0.0100
10	0.0292	0.0292	0.0004	0.0000	0.1336	0.0097
13	0.0271	0.0271	0.0004	0.0000	0.1260	0.0098
15	0.0251	0.0250	0.0004	0.0001	0.5487	0.0099
20	0.0210	0.0210	0.0004	0.0001	0.2882	0.0105
25	0.0179	0.0178	0.0003	0.0001	0.7801	0.0112
30	0.0154	0.0153	0.0003	0.0001	0.8337	0.0119
40	0.0120	0.0119	0.0003	0.0001	0.9063	0.0133
50	0.0099	0.0096	0.0003	0.0003	2,9442	0,0145

In graphic 3, are displayed the results of the values of the geometric factor reported by Conway J.[3](dot line) and the obtained by the ESGF method (solid line), foraxial circular detector-circumferential sourcesystems, corresponding to the following parameters: (a) $h=2$, $r_F=5$; (b) $h=0.01$, $r_F=10$; (c) $r_D=3$; (d) $r_D=1$, $r_F=2$.

In the graphic 4, it is shown the comparison of values of the geometric factors for non-axial circular detector-circumferential source systems obtained by Conway J. [3] (cross) and the ESGF method (solid line). The graphic 5 corresponds to the following parameters: (a) $\boldsymbol{h}=2$, $\boldsymbol{r_D}=3$, $\boldsymbol{r_F}=5$; (b) $\boldsymbol{r_D}=2$, $\boldsymbol{r_F}=3$, $\boldsymbol{d}=1$; (c) $\boldsymbol{r_D}=2$, $\boldsymbol{r_F}=3$, $\boldsymbol{d}=3$; (d) $\boldsymbol{r_D}=2$, $\boldsymbol{r_F}=3$, $\boldsymbol{d}=5$.

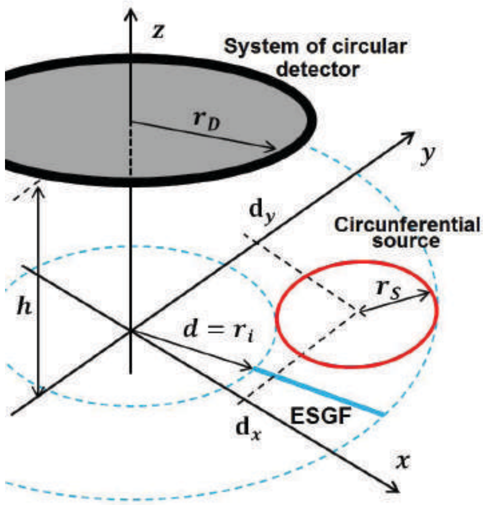
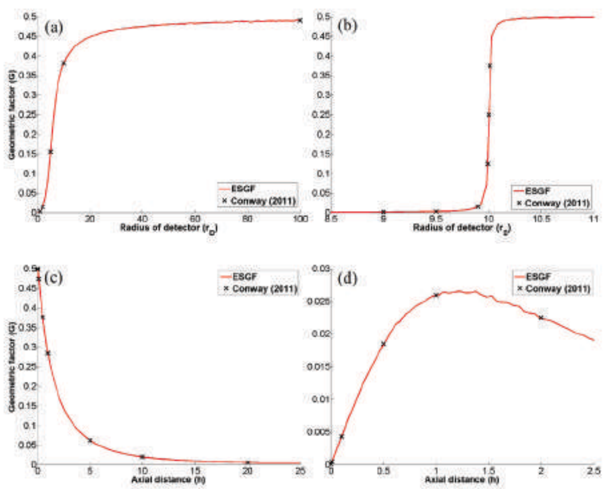


Figure 7: Circular detector – circumferential source parallel system, of radius $\boldsymbol{r_S}$ (source's radius), and $\boldsymbol{r_D}$ (detector's radius), with separation between planes of h, and displacement of the source, respect to the detector's symmetry axis of $\boldsymbol{d}$.



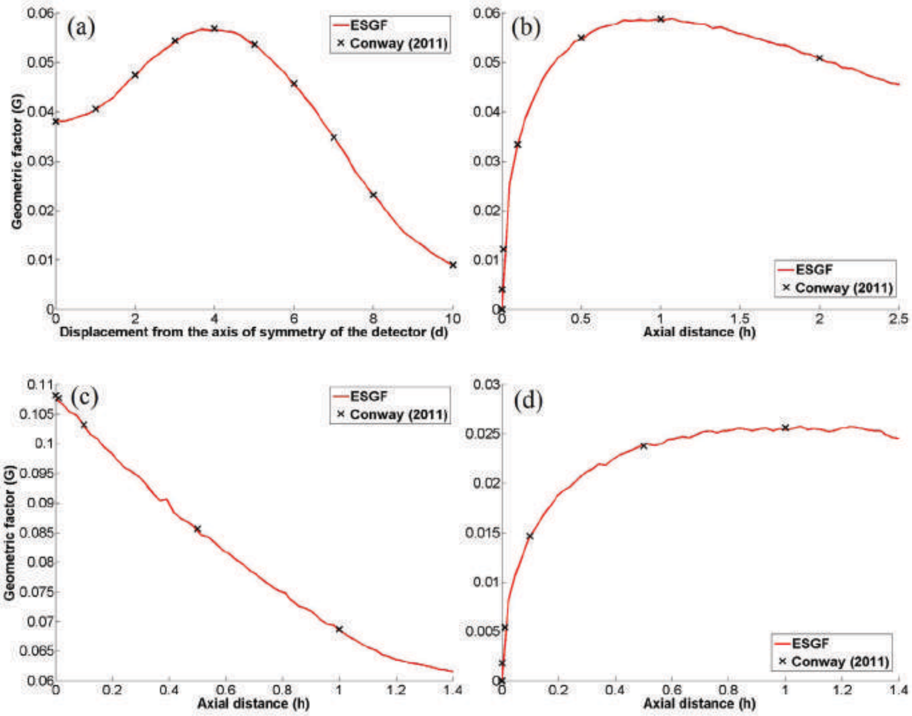
Graphic 3: Geometric efficiency for a circular detector of radius $\boldsymbol{r_D} = 5$, and a homogeneous linear sources (dot line) at an axial distance from the detector of $\boldsymbol{h} = 10$, displaced from the symmetry axis (a) $\boldsymbol{a_T} = 0$, $\boldsymbol{a_L} = 0$, (b) $\boldsymbol{a_T} = 0$, $\boldsymbol{a_L} = 5$, (c) $\boldsymbol{a_T} = 5$, $\boldsymbol{a_L} = 0$, (d) $\boldsymbol{a_T} = 5$, $\boldsymbol{a_L} = 5$, as a function of its semi-length, and the corresponding non-homogeneous linear ESGF of radial orientation (solid line).

Table 2: Values of the geometric factor obtained by Conway J. [3] $\left(G_p\right)$ and for the ESGF method $\left(G_s\right)$, corresponding to the curve (b), (c) and (d) of graphic 5. Confidence Interval $\left(CI\right)$, absolute error (ε_A), relative error ($\varepsilon_{\%}$) and relative standard deviation (ε_S).

	h	G_s	G_p	I	ε_A	$\varepsilon_{\%}$	ε_S
$r_D=2$ $r_F=3$ $d=1$	0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	0,001	0,0039	0,0040	0,0001	0,0001	2,0505	0,0311
	0,01	0,0123	0,0122	0,0002	0,0001	0,5347	0,0176
	0,1	0,0335	0,0334	0,0004	0,0001	0,2422	0,0105
	0,5	0,0547	0,0550	0,0004	0,0003	0,4552	0,0081
	1	0,0587	0,0587	0,0005	0,0000	0,0831	0,0078
	2	0,0509	0,0509	0,0004	0,0000	0,0105	0,0085
$r_D=2$ $r_F=3$ $d=3$	0	0,1079	0,1082	0,0006	0,0003	0,2583	0,0056
	0,001	0,1083	0,1081	0,0006	0,0002	0,1865	0,0056
	0,01	0,1073	0,1077	0,0006	0,0004	0,3347	0,0057
	0,1	0,1031	0,1032	0,0006	0,0001	0,1153	0,0058
	0,5	0,0852	0,0856	0,0005	0,0004	0,4617	0,0064
	1	0,0681	0,0687	0,0005	0,0006	0,9429	0,0073
$r_D=2$ $r_F=3$ $d=5$	0	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
	0,001	0,0017	0,0018	0,0001	0,0001	4,9776	0,0474
	0,01	0,0053	0,0054	0,0001	0,0001	1,9479	0,0268
	0,1	0,0147	0,0146	0,0002	0,0000	0,2310	0,0161
	0,5	0,0236	0,0238	0,0003	0,0002	0,6676	0,0126
	1	0,0254	0,0256	0,0003	0,0002	0,8011	0,0121

The number of stories for obtaining values table 3 was $N = 10^6$, with a mean machine time $\overline{t_S} = 0{,}92$ seg.

Circular source. The validation of the equivalence between the geometric efficiency of a homogenous circular source of radius $r_F = 1$ coaxially placed in different parallel planes at a distances $h = 0$, $h = 1$, $h = 2$, $h = 3$, $h = 4$ and $h = 5$ from a circular detector of radius r_D, and an inhomogeneous radially oriented linear ESGF, was conducted with reference to the equation of Ruby et al. [12].



Graphic 4: Geometric factor for non-axial circular detector-circumferential source systems, obtained by Conway J. [3] (dot line) and by the ESGF method (solid line). (a) $\boldsymbol{h} = 2$, $\boldsymbol{r_D} = 3$, $\boldsymbol{r_S} = 5$; (b) $\boldsymbol{r_D} = 2$, $\boldsymbol{r_S} = 3$, $\boldsymbol{d} = 1$; (c) $\boldsymbol{r_D} = 2$, $\boldsymbol{r_S} = 3$, $\boldsymbol{d} = 3$; (d) $\boldsymbol{r_D} = 2$, $\boldsymbol{r_S} = 3$, $\boldsymbol{d} = 5$.

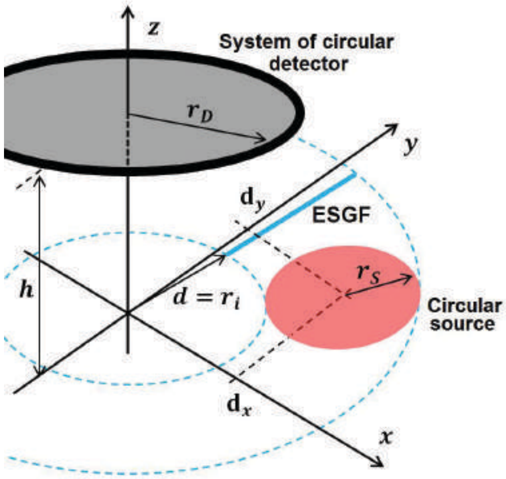
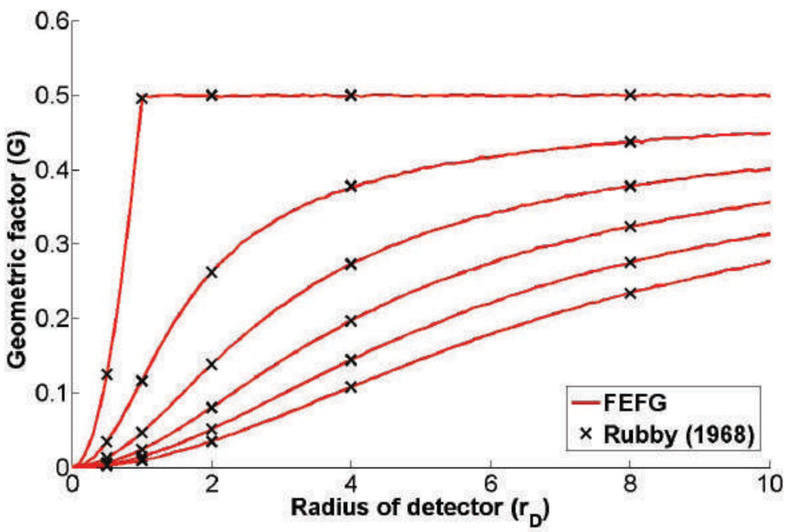


Figure 8: Circular detector – circular source parallel system, of radius r_S (source's radius), and r_D (detector's radius), with separation between planes of h, and displacement of the source, respect to the detector's symmetry axis of d .



Graphic 5: Geometric factor of coaxial circular detector- source systems, obtained by Ruby et al. [12] (cross) and by the ESGF method (solid line) for source radius $r_S = 1$, and different axial distances from the detector (a) $h = 0$, (b) $h = 1$, (c) $h = 2$, (d) $h = 3$, (e) $h = 4$ and (f) $h = 5$.

Table 3: Values of the geometric factor obtained by Ruby et al [12] (***G_p***) and for the ESGF method (***G_s***), corresponding to the curve "a" of graphic 5. Confidence Interval (***CI***), absolute error (ε_A), relative error ($\varepsilon_{\%}$) and relative standard deviation (ε_S).

r_D	G_s	G_p	I	ε_A	$\varepsilon_{\%}$	ε_S
0.5	0.0024	0.0024	0.0008	0.0000	0.7260	0.0011
1,0	0.0095	0.0094	0.0049	0.0001	0.8684	0.0016
2,0	0.0350	0.0350	0.0208	0.0000	0.1000	0.0019
4,0	0.1083	0.1082	0.0666	0.0001	0.1228	0.0019
8,0	0.2341	0.2342	0.1453	0.0001	0.0587	0.0019

6. DISCUSSION AND CONCLUSIONS

Given the importance of the knowledge of the geometric factor, especially when absolute measurements of the intensity of the emitting materials of ionizing radiations, including the detectors calibration for absolute methods are required, it have been developed various methods where the use of the Monte Carlo method has demonstrated its applicability to calculate the geometric factor of circular detection systems, and point, linear, circumferential, and circular sources, among others. Most equations proposed are of such a complexity that is preferred to use approaches which results sometimes have uncertainties greater than the acceptable.

In this study, a new method applicable to planar sources parallel to a circular detector was developed, through which is possible to obtain infinite sources with different geometries but the same geometric factor (ESGF).

Depending on the details required in the description of the physical phenomenon, geometric patterns, and the characteristics of the radiation sources, Monte Carlo calculations may, in particular cases, very demanding in terms of computational resources. The azimuthal invariance of the solid angle of point sources located in a plane perpendicular to the axis of symmetry of a circular detector equidistant from the axis, allows us to obtain ESGF sources of lower dimensions than the source under study, resulting in an algorithm relatively simpler and less machine time when calculating the geometric factor by Monte Carlo method and simplified analytical calculations, if applicable. When comparing the values of the geometric factors of the ESGF corresponding to some published in the literature, it is found a very good agreement; indicating that this method is suitable and convenient for obtaining sources with multiple geometries and identical geometric factor.

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