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Quantum Effects on the Linear Dispersion Characteristics in Electron-Positron Plasma

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Abstract: In electron-positron plasmas some of the plasma modes are decoupled due to the equal charge to mass ratio of both species. The dispersion properties of the propagation of linear waves in degenerate electron–positron magnetoplasma are investigated. By using the quantum hydrodynamic equations with magnetic fields of the Wigner–Maxwell system, we have obtained a set of new dispersion relations in which ions' motions are not considered. The general dielectric tensor is derived using the electron and positron densities and its momentum response to the quantum effects due to Bohm potential and the statistical effect of Femi temperature. It has been demonstrated the importance of magnetic field and its role with the quantum effects in these plasmas which support the propagation of electromagnetic linear waves. Besides, the dispersion relations in case of parallel and perpendicular modes are investigated for different positron-electron density ratios.

Keywords: Quantum Plasma; Dispersion relation ; Electron–Positron

1. INTRODUCTION

Electron-positron (e-p) plasmas are found in the early universe[1,2], in astrophysical objects (e.g., pulsars[3], super nova remnants, and active galactic nuclei[4,5], in γ -ray bursts[6], and at the center of the Milky Way galaxy [7].

In such physical systems, the e-p pairs can be created by collisions between particles that are accelerated by electromagnetic and electrostatic waves and/ or by gravitational forces. Intense laser-plasma interaction experiments have reported the production of MeV electrons and conclusive evidence of positron

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production via electron collisions. Positrons have also been created in post disruption plasmas in large tokamaks through collisions between MeV electrons and thermal particles. The progress in the production of positron plasmas of the past two decades makes it possible to consider laboratory experiments on e-p plasmas [8].

The earlier theoretical studies on linear waves in electron–positron plasmas have largely focused on the relativistic regime relevant to astrophysical contexts [9-12]. This is largely due to the fact that the production of these electron–positron pairs requires high-energy processes. In laboratory plasmas non-relativistic electron–positron plasmas can be created by using two different schemes. In one scheme, a relativistic electron beam when impinges on high Z-target produces positrons in abundance. The relativistic pair of electrons and positrons is then trapped in a magnetic mirror and cools down rapidly by radiation, thus producing non-relativistic pair plasmas. In another scheme positrons can be accumulated from a radioactive source. Such non-relativistic electron–positron plasmas have been produced in the laboratory by many researchers.

This has given an impetus to many theoretical works on non-relativistic electron–positron plasmas. Stewart and Laing [13] studied the dispersion properties of linear waves in equal-mass plasmas and found that due to the special symmetry of such plasmas, well known phenomena such as Faraday rotation and whistler wave modes disappear. Iwamoto [14] studied the collective modes in non-relativistic electron–positron plasmas using the kinetic approach. He found that the dispersion relations for longitudinal modes in electron–positron plasma for both unmagnetized and magnetized electron–positron plasmas were similar to the modes in one-component electron or electron–ion plasmas. The transverse modes for the unmagnetized case were also found to be similar. However, the transverse modes in the presence of a magnetic field were found to be different from those in electron–ion plasmas. Studies of wave propagation in electron–positron plasmas continue to highlight the role played by the equal mass of electrons and positrons. For example, the low frequency ion acoustic wave, a feature of electron–ion plasmas due to significantly different masses of electrons and ions, has no counterpart in electron–positron plasma. Shukla et al [15] derived a new dispersion relation for low-frequency electrostatic waves in strongly magnetized non-uniform electron–positron plasma. They showed that the dispersion relation admits a new purely growing instability in the presence of equilibrium density and magnetic field inhomogeneties. Linear electrostatic waves in a magnetized four-component, two-temperature electron–positron plasma are investigated by Lazarus et al in Ref. [16]. They have derived a linear dispersion relation for electrostatic waves for the model and analyzed for different wave modes. Dispersion characteristics of these modes at different propagation angles are studied numerically.

In this work, The dispersion properties of the propagation of linear waves in degenerate electron–positron magnetoplasma are investigated. By using the quantum hydrodynamic equations with magnetic fields of the Wigner–Maxwell system, we have obtained a set of new dispersion relations in which ions' motions are not considered. The general dielectric tensor is derived using the electron and positron densities and its momentum response to the quantum effects due to Bohm potential and the statistical effect of Femi temperature.

2. MODELING EQUATIONS

We consider quantum plasma composed of electrons and positrons whose background stationary ions. The plasma is immersed in an external magnetic field $\vec{B}_0 = B_0 \hat{e}_z$. The quasi-neutrality condition reads as $n_{io} + n_{po} = n_{eo}$. From $n_\alpha, \vec{u}_\alpha$ model, the dynamics of these particles are governed by the following continuity equation and the momentum equation:

$$\frac{\partial n_\alpha}{\partial t} + \nabla. (n_\alpha \vec{u}_\alpha) = 0, \tag{1}$$

$$m_\alpha \frac{\partial \vec{u}_\alpha}{\partial t} = q(\vec{E} + \vec{u}_\alpha \times \vec{B}) - \frac{\nabla P_\alpha}{n_\alpha} + \frac{\hbar^2}{2m_\alpha} \nabla\left(\frac{1}{\sqrt{n_\alpha}} \nabla^2 \sqrt{n_\alpha}\right) \tag{2}$$

Here $n_\alpha, \vec{u}_\alpha$ and m_α are the number density, the velocity and the mass of $\alpha-$ particle respectively ($\alpha = e, p$) and $\hbar$ is the plank constant divided by 2π . Let electrons and positrons obey the following pressure law:

$$P_\alpha = \frac{mV_{F\alpha}^2}{3n_{\alpha 0}} n_\alpha$$

Where, $V_{F\alpha} == \sqrt{\frac{2k_B T_{F\alpha}}{m_\alpha}}$ is the Fermi thermal speed, $T_{F\alpha}$ is the particle Fermi temperature, K_B is the Boltzmann's constant and $n_{\alpha o}$ is the equilibrium particle number density. We have included both the quantum statistical effects through Fermi temperature and the quantum diffraction in the $\hbar$ –dependent. If we set $\hbar$ equal to zero and $T_{F\alpha}$ equal the temperature of electrons and positrons, we obtain the classical hydrodynamic equation. Assuming that the plasma is isothermal, the Fermi speeds for different particles may be equal.

Using the perturbation technique, assume the quantity ϕ representing (n, u, B, E) has the following form $\phi = \phi_0 + \phi_1$ where ϕ_0 is the unperturbed value and $\phi_1 << \phi_0$ is a small perturbation ϕ $\phi \propto \exp(-i\omega t + i\vec{k}\cdot\vec{r})$. Assuming the equilibrium electric field is zero and linearizing the continuity and the momentum equations, we have:

$$n_{\alpha 1} = \frac{\vec{k}\cdot\vec{u}_{\alpha 1}}{\omega - \vec{k}\cdot\vec{u}_{\alpha 0}} n_{\alpha 0}\,, \tag{3}$$

$$-i\omega\vec{u}_{\alpha 1} = \frac{q_\alpha}{m_\alpha}\left(\vec{E} + \vec{u}_{\alpha 0}\times\vec{B} + \vec{u}_{\alpha 1}\times\vec{B}_0\right) - \frac{iQ_\alpha(\vec{k}\cdot\vec{u}_{\alpha 1})}{(\omega - \vec{k}\cdot\vec{u}_{\alpha 0})}\vec{k} \tag{4}$$

Multiplying equation (4) by $\vec{k}$ and Simplifying, we can obtain the following equation:

$$\vec{u}_{\alpha 1} = \frac{iq_\alpha}{m_\alpha\omega}\left(\vec{g} + \vec{u}_{\alpha 1}\times\vec{B}_0\right) + \frac{iq_\alpha}{m_\alpha\omega}\frac{Q_\alpha\vec{k}}{\Omega_\alpha(\omega - \vec{k}\cdot\vec{u}_{\alpha 0})}\left[\left(\vec{g} + \vec{u}_{\alpha 1}\times\vec{B}_0\right)\cdot\vec{k}\right] \tag{5}$$

where,

$$\Omega_\alpha = \omega - \frac{Q_\alpha k^2}{\omega_\alpha'}, \quad \omega_\alpha' = \omega - \vec{k}\cdot\vec{u}_{\alpha 0}\,,$$

$$Q_\alpha = V_{F\alpha}^2 + \frac{\hbar^2 k^2}{4m_\alpha^2}, \text{ and } \vec{g} = \frac{1}{\omega}\left[(\omega - \vec{k}\cdot\vec{u}_{\alpha 0})\vec{E} + (\vec{u}_{\alpha 0}\cdot\vec{E})\vec{k}\right]$$

Assuming, $\vec{u}_{\alpha 0} = u_{\alpha 0}\vec{e}_z$, $\vec{B}_0 = B_0\vec{e}_z$, $\vec{k} = (k_x, 0, k_z)$, then the three components of the fluid velocity can be written as:

$$\begin{aligned} u_{\alpha 1x} = \frac{1}{F}\Bigg\{ & \frac{iq_\alpha G}{m_\alpha\omega^2}(\omega - k_z u_{\alpha 0})E_x - \frac{q_\alpha\omega_{c\alpha}^2}{\omega^3}\frac{G}{m_\alpha}E_y \\ & + \frac{iq_\alpha k_x}{m_\alpha\omega^2}\left[u_{\alpha 0} + \frac{Q_\alpha}{\Omega_\alpha}\left(k_z + \frac{k^2 u_{\alpha 0}}{(\omega - k_z u_{\alpha 0})}\right)\right]E_z\Bigg\} \end{aligned} \tag{6a}$$

$$\begin{aligned} u_{\alpha 1y} = \frac{1}{F}\Bigg\{ & \frac{q_\alpha(\omega - k_z u_{\alpha 0})\omega_{c\alpha}G}{m_\alpha\omega^3}E_x - \frac{iq_\alpha}{m_\alpha}\frac{(\omega - k_z u_{\alpha 0})}{\omega^2}E_y \\ & + \frac{q_\alpha k_x\omega_{c\alpha}}{m_\alpha\omega^3}\left[u_{\alpha 0} + \frac{Q_\alpha}{\Omega_\alpha}\left(k_z + \frac{k^2 u_{\alpha 0}}{(\omega - k_z u_{\alpha 0})}\right)\right]E_z\Bigg\} \end{aligned} \tag{6b}$$

$$u_{\alpha 1z} = \frac{iq_\alpha k_x k_z}{m_\alpha \omega^2}\frac{Q_\alpha}{\Omega_\alpha F}\left[E_x + \frac{i\omega_{c\alpha}}{\omega}E_y\right] + \frac{iq_\alpha}{m_\alpha \omega^2}\left\{\omega + \frac{Q_\alpha k_z}{\Omega_\alpha}\left(k_z + k^2 u_{\alpha 0}\right) - \frac{k_x^2 k_z Q_\alpha}{\Omega_\alpha(\omega - k_z u_{\alpha 0})F}\left(\frac{\omega_{c\alpha}^2}{\omega^2}\right)\left[u_{\alpha 0}\left(1 + \frac{k^2 Q_\alpha}{\Omega_\alpha(\omega - k_z u_{\alpha 0})}\right) + \frac{Q_\alpha k_z}{\Omega_\alpha}\right]\right\}E_z \tag{6c}$$

Where,

$$G = 1 + \frac{Q_\alpha k_x^2}{\Omega_\alpha(\omega - k_z u_{\alpha 0})} \text{ and } F = 1 - \frac{\omega_{c\alpha}^2}{\omega^2}G$$

The current density and the dielectric permeability of the medium are given:

$$\vec{J} = \sum_\alpha\left(q_\alpha n_{\alpha 0}\vec{u}_{\alpha 1} + q_\alpha n_{\alpha 1}\vec{u}_{\alpha 0}\right) = \hat{\sigma}\cdot\vec{E} \tag{7}$$

$$\hat{\varepsilon} = \hat{I} - \frac{i}{\epsilon_0\,\omega}\hat{\sigma} \tag{8}$$

where $\hat{I}$ is the unit tensor. So, we can obtain the dielectric tensor as follows:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} \tag{9}$$

Where,

$$\varepsilon_{11} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_\alpha^{/}}{\omega^3}\left(\frac{G}{F}\right)$$

$$\varepsilon_{12} = \sum_\alpha \frac{-i\omega_{p\alpha}^2\,\omega_{c\alpha}^2}{\omega^4}\left(\frac{G}{F}\right)$$

$$\varepsilon_{13} = \sum_\alpha \frac{-\omega_{p\alpha}^2\,k_x}{\omega^3 F}\left[u_{\alpha 0} + \frac{Q_\alpha}{\Omega_\alpha}\left(k_z + \frac{k^2 u_{\alpha 0}}{\omega_\alpha^{/}}\right)\right]$$

$$\varepsilon_{22} = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2 \omega_\alpha^/}{\omega^3 F}$$

$$\varepsilon_{31} = \sum_{\alpha} \frac{-\omega_{p\alpha}^2 k_x Q}{\omega \omega_\alpha^/ F} \left[u_{\alpha 0} + \frac{Q_\alpha k_z}{\Omega_\alpha} \right]$$

$$\varepsilon_{32} = \sum_{\alpha} \frac{-i\omega_{p\alpha}^2 k_x \omega_{c\alpha}}{\omega^2 \omega_\alpha^/ F} \left[\frac{u_{\alpha 0}\omega_{c\alpha} G}{\omega} + \frac{Q_\alpha k_z}{\Omega_\alpha} \right]$$

$$\varepsilon_{33} = 1 - \sum_{\alpha} \left(\frac{\omega_{p\alpha}^2}{\omega \omega_\alpha^/} \right) \left[1 + \frac{k_x^2 u_{\alpha 0}^2}{\omega^2 F} - \frac{k_x^2 k_z \omega_{c\alpha}^2 Q}{\omega \omega_\alpha^/ \Omega_\alpha^2 F} \left(u_{\alpha 0} - \frac{Q k_z}{\omega} \right) \right.$$
$$\left. + \frac{Q_\alpha k_z^2}{\omega \Omega_\alpha} \left(1 + \frac{k_x^2 u_{\alpha 0}}{k_z \omega F} \right) \left(1 + \frac{k^2 u_{\alpha 0}}{k_z \omega_\alpha^/} \right) \right]$$

Then, according to equations (8), (9) The propagation of different electromagnetic linear waves in quantum plasma can be obtained from the following general dispersion relation:

$$Det \begin{pmatrix} \varepsilon_{11} - N_z^2 & \varepsilon_{12} & \varepsilon_{13} + N_x N_z \\ \varepsilon_{21} & \varepsilon_{22} - N^2 & \varepsilon_{23} \\ \varepsilon_{31} + N_x N_z & \varepsilon_{32} & \varepsilon_{33} - N_x^2 \end{pmatrix} = 0 \qquad (10)$$

Where, $\omega_{p\alpha} = \left(n_{\alpha 0} q_\alpha^2 / m_\alpha \in_0 \right)^{1/2}$ is the plasma frequency and $N_{x,z} = c\, k_{x,z} / \omega$.

3. DISCUSSION

In this section, we focus our attention on the discussion of some different modes in two cases that the wave vector parallel and perpendicular to the magnetic field $\left(\vec{k} \, / / \, and \perp \vec{B}_0 \right)$.

(3.1) Parallel modes $\left(k_x = 0 \, , \; k = k_z \right)$:

So, this case leads to, $G = 1, \quad F = 1 - \frac{\omega_{c\alpha}^2}{\omega^2}$ with

$$\varepsilon_{13} = \varepsilon_{31} = \varepsilon_{23} = \varepsilon_{32} = 0$$

$$\varepsilon_{11} = \varepsilon_{22} = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2 (\omega - k_z u_{\alpha 0})}{\omega(\omega^2 - \omega_{c\alpha}^2)}$$

$$\varepsilon_{12} = \sum_{\alpha} \frac{-i\omega_{p\alpha}^2 \omega_{c\alpha}^2}{\omega^2(\omega^2 - \omega_{c\alpha}^2)}$$

$$\varepsilon_{21} = \sum_{\alpha} \frac{i\omega_{p\alpha}^2 \omega_{c\alpha}(\omega - k_z u_{\alpha 0})}{\omega^2(\omega^2 - \omega_{c\alpha}^2)}$$

$$\varepsilon_{33} = 1 - \sum_{\alpha} \frac{\omega_p^2}{\omega(\omega - k_z u_{\alpha 0})}\left[1 + \frac{k_z^2 Q_\alpha}{\omega \Omega_\alpha}\left(1 + \frac{k_z u_{\alpha 0}}{(\omega - k_z u_{\alpha 0})}\right)\right]$$

Therefore the general dispersion relation (10) becomes:

$$\varepsilon_{33}\left[(\varepsilon_{11} - N_z^2)^2 - \varepsilon_{21}\varepsilon_{12}\right] = 0 \tag{11}$$

This gives two dispersion relations. The first one ($\varepsilon_{33} = 0$) investigates the dispersion of electrostatic quantum waves included the quantum effects as follows

$$1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega \omega_\alpha^{/}}\left[1 + \frac{k_z^2 Q_\alpha}{\omega \Omega_\alpha}\left(1 + \frac{k_z u_{\alpha 0}}{\omega_\alpha^{/}}\right)\right] = 0 \tag{12}$$

By neglecting the quantum effects, equation (11) describes the following well-known classical modes

$$1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega \omega_\alpha^{/}} = 0$$

The second dispersion equation gives:

$$1 - \frac{c^2 k_z^2}{\omega^2} - \sum_{\alpha}\left[\frac{\omega_{p\alpha}^2 \omega_\alpha^{/}}{\omega^3 F} \pm \frac{\omega_{p\alpha}^2 \omega_{c\alpha}}{\omega^4 F}\sqrt{\omega_{c\alpha}\omega_\alpha^{/}}\right] = 0 \tag{13}$$

Equation (13) is similar to the dispersion of left and right waves (L- and R- modes). Owing to the symmetry between the positively and negatively charged particles, the dispersion relation for the right circularly polarized wave is identical to the left circularly polarized wave. It has been noted that no quantum effects on these modes. For unmagnetized plasma , the dispersion relation becomes:

$$\omega^3 - \sum_{\alpha}\left[\omega(\omega_{p\alpha}^2 + c^2 k_z^2) - \omega_{p\alpha}^2 k_z u_{\alpha 0}\right] = 0 \tag{14}$$

(3.2) Perpendicular mode $\left(k_z = 0\,,\ k = k_x\right)$:

In this case, we have

$$\omega'_\alpha = \omega\,, \quad \Omega_\alpha = \omega - \frac{k_x^2 Q_\alpha}{\omega}$$

So, the general dispersion relation (10) becomes:

$$Det\begin{pmatrix} \varepsilon_{11} & -i\,\varepsilon_{12} & -\,\varepsilon_{13} \\ i\,\varepsilon_{21} & \varepsilon_{22} - N_x^2 & i\,\varepsilon_{23} \\ -\,\varepsilon_{31} & -i\varepsilon_{32} & \varepsilon_{33} - N_x^2 \end{pmatrix} = 0 \tag{15}$$

Where it has the following new elements

$$\varepsilon_{11} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha} \qquad \varepsilon_{22} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2 (\omega^2 - k_x^2 Q_\alpha)}{\omega^2 (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)}$$

$$\varepsilon_{12} = \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_{c\alpha}^2}{\omega^2 (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)} \qquad \varepsilon_{21} = \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_{c\alpha}}{\omega (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)}$$

$$\varepsilon_{13} = \varepsilon_{31} = \sum_\alpha \frac{\omega_{p\alpha}^2 k_\alpha u_\alpha}{\omega (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)} \qquad \varepsilon_{23} = \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_{c\alpha} k_\alpha u_\alpha}{\omega^2 (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)}$$

$$\varepsilon_{32} = \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_{c\alpha}^2 k_\alpha u_\alpha}{\omega^3 (\omega^2 - \omega_{c\alpha}^2 - k_x^2 Q_\alpha)} \qquad \varepsilon_{33} = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\left[1 - \frac{k_x^2 u_\alpha^2}{\omega^2 - k_x^2 Q_\alpha}\right]$$

In the case of unmagnetized plasma $(\omega_{c\alpha} = 0)$, we have the following two dispersion equations:

$$\omega^2 = \omega_{pe}^2 + \omega_{pp}^2 + c^2 k_x^2 \tag{16}$$

and

$$\left[1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - k_x^2 Q_\alpha}\right]\left[1 - N_x^2 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2}\left(1 - \frac{k_x^2 u_\alpha^2}{\omega^2 - k_x^2 Q_\alpha}\right)\right] - \sum_\alpha \frac{\omega_{p\alpha}^4 k_x^2 u_\alpha^2}{\omega^2 (\omega^2 - k_x^2 Q_\alpha)^2} = 0 \tag{17}$$

The equation (16) is the well known dispersion relation which investigates the propagation of electromagnetic waves in classical unmagnetized plasma. The damping is absent because the phase velocity of the wave obtained from this equation is always greater than the velocity of light, so that no particles can be resonant with the wave. This results is analogous to the one-component electron plasma [14]. While the other relation (17) indicates the dispersion of the waves in electron-positron plasma under the quantum effects.

4. NUMERICAL ANALYSIS AND RESULTS

In this section, we are going to investigate the above dispersion relations numerically. Introducing the normalized quantities $\bar{\omega}_c = \omega_c / \omega_p$, $\gamma = \omega_{pp} / \omega_{pe}$ $V = u_{eo} / V_{Fe}$, $U = u_{po} / u_{eo}$ and the plasmonic coupling $(H = \hbar\omega_p / 2mV_{Fe}^2)$ which describes the ratio of plasmonic energy density to the electron Fermi energy density, we rewrite some of the dispersion relations in both of parallel and perpendicular modes.

(4.1) Parallel modes

In the first, equation (12), ($\varepsilon_{33} = 0$) becomes:

$$\begin{aligned} &1 - W(W-KV) + \frac{AK^2(W-KU)}{W\left(W(W-KV) - AK^2\right)} \\ &- \frac{\gamma^2(W-KV)}{W-KU} - \frac{AK^2\gamma^2(W-KV)}{W\left(W(W-KU) - AK^2\right)} = 0 \end{aligned} \tag{18}$$

Where, $A = 1 + H^2$.

The dispersion relation (17) has two positive solutions, Fig 1, for positron electron density ration $(\gamma = 0.5)$ with $(H = 2)$ and $(U = 1)$.One of solutions of the dispersion equation (19) can be investigated in Fig. (2) to study the parallel modes for different density ratios $(\gamma = 0.1, 0.5, 0.7, 0.9)$ with $(U = 1)$ in quantum plasma $(H = 2)$.

The solution of the normalized dispersion equation (17) has been also displayed in 3D figure (3) for quantum unmagnetized plasma $(H = 4)$.

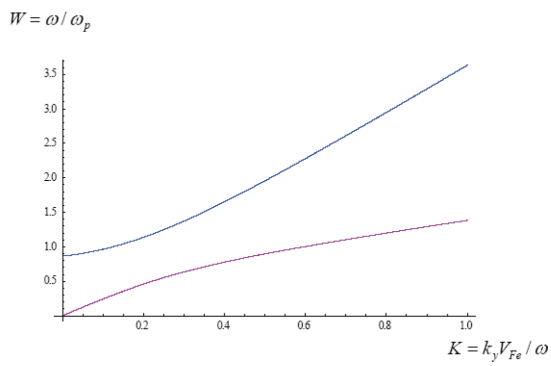

Figure 1: The dispersion relation (5.19) has two positive solutions for positron electron density ration (γ = 0.5) with (H = 2) and (U = 1)

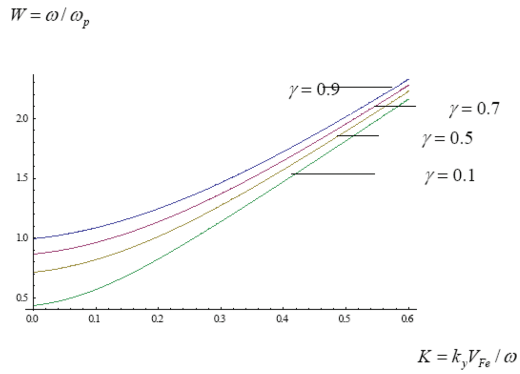

Figure 2: The dispersion relations of the modes for different density positron-electron ratios (γ = 0.1, 0.5, 0.7, 0.9) with (H = 2) and (U = 1)

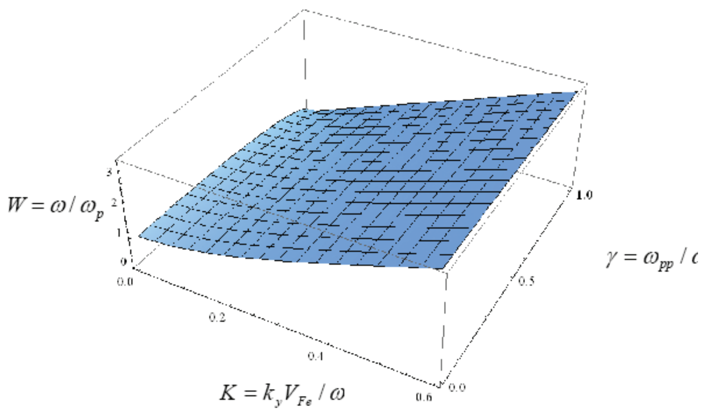

Figure 3: The dispersion relations of the parallel modes along density ratio γ −axis with (H = 2) and (U = 1)

It is clear from the previous figures that the dispersion relations depend strongly on the density ratio of positron to electron. As the positron density is increased to equal to the electron density, the phase velocity has been increased. In the beginning, with very small positron density the wave frequency equals the electron plasma frequency and decreased with positron density increased.

Besides, in the Fig. (4), the dispersion relation of parallel modes is shown for different quantum ratios $(H = 0,1,2,4)$, in the case of positron-electron density ratio $(\gamma = 0.9)$ and equal velocities of them (U = 1). It is clear that the phase velocity of the mode is increased with the increases of plasmonic coupling ratio.

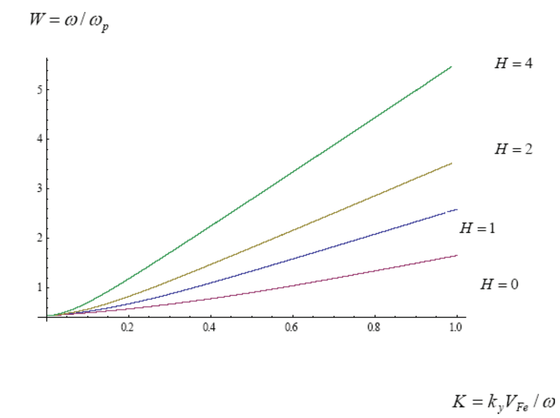

Figure 4: The dispersion relations of different modes for different quantum effects (H = 0,1,2,4) with positron-electron density ratio (γ = 0.9) and velocity ratio (U = 1).

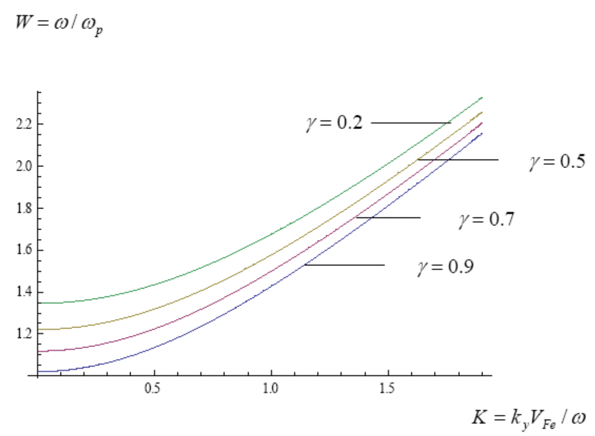

Figure 5: The dispersion relations of electromagnetic modes for different ratios (γ = 0.2, 0.5, 0.7, 0.9) in classical plasma.

(4.2) Perpendicular mode

In the case of perpendicular modes, equation (15) can be normalized and solved numerically (here, $K=ck_y/\omega_p$). Figure (5) displays the dispersion curves of electromagnetic modes under the effect of different density ratios ($\gamma=$ 0.2, 0.5, 0.7. 0.9) in classical plasma.

Also, the other equation (16) can be solve numerically to give two real solutions. One of them is the same solution approximately of equation (15) (which is clear in Figure (6). The other solution of dispersion equation (16) is displayed in figure (7).

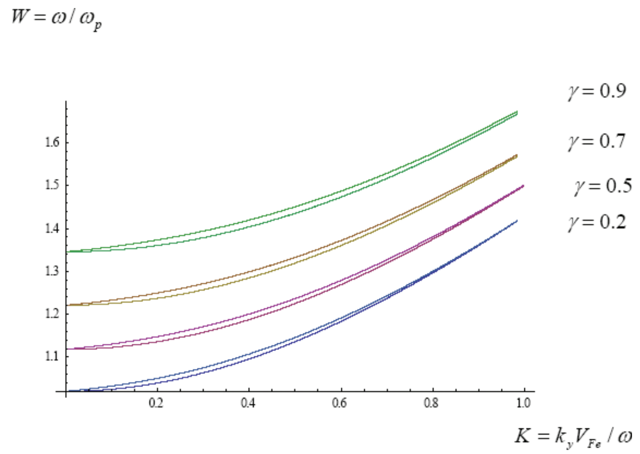

Figure 6: The dispersion solutions of the equations (5.17) and (5.18) for different density ratios ($\gamma = 0.2, 0.5, 0.7, 0.9$).

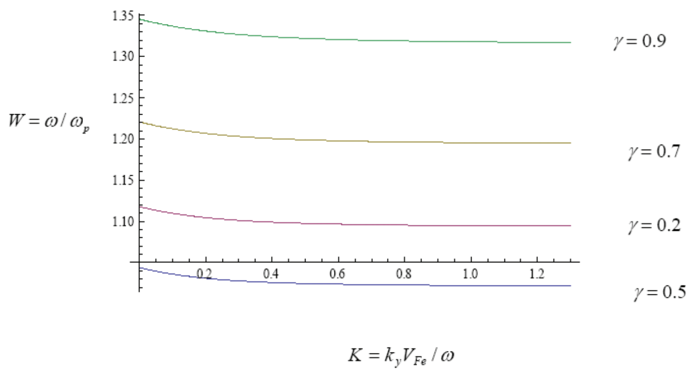

Figure 7: The other dispersion solutions of the equation (18) for different density ratios ($\gamma = 0.2, 0.5, 0.7, 0.9$).

It is clear in the figures that the dispersion curves at $K = c\,k_y / \omega_p = 0$ depend essentially on the positron-electron density ratio $(\gamma = 0.1, 0.5, 0.7. 0.9)$. As the positron density increases to equal electron density, the wave frequency is increased to be bigger than the plasma frequency.

On the dispersion curves (figures (5) and (6)), it has been noted the phase velocity of modes (+ve slope of the curves) decreases as density ratio increases. But, on the figure (7), the phase velocities of these modes (-ve slope) are the same with changes of the density ratio. They tend to zero with large wave number which means that these modes cannot propagate in plasmas.

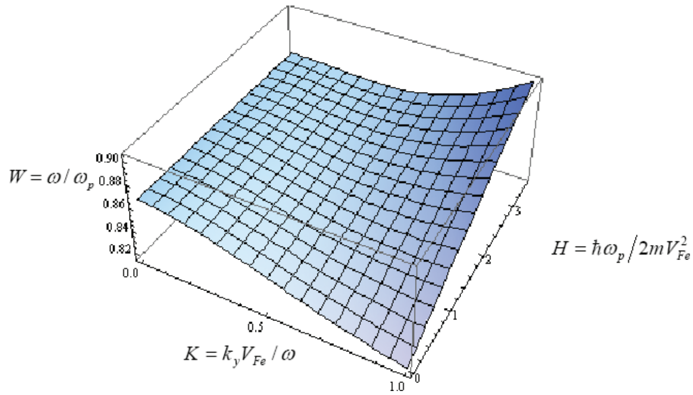

Figure 8: 3D plotting for dispersion relation for perpendicular modes in quantum unmagnetized plasma along quantum ratio axis with ($\gamma = 0.5$, $V = 0.2$)

Figure (8) investigates the dispersion relations of the electromagnetic waves in electron-positron plasma ($\gamma = 0.5$) under the quantum effects. It is clear that, in the case of classical plasma, the wave frequency decreases as wave number increases (the phase velocity is negative). But, in the case of quantum plasma (for small ratio *H*), the wave frequency deceases as wave number increases (the phase velocity is negative). Then, the phase velocity and group velocity tends to zero at definite wave number (*K*) depends on the quantum ratio (*H*). For high quantum ratio, the phase velocity starts to be +ve and increases again.

5. CONCLOUSION

In this work, The dispersion properties of the propagation of linear waves in degenerate electron–positron magnetoplasma are investigated by using the quantum hydrodynamic equations with magnetic fields of the Wigner–Maxwell system. The general dielectric tensor is derived using the electron and positron densities and its momentum response to the quantum effects due to Bohm potential and the statistical effect of Femi temperature. We have obtained a set of new dispersion relations in two cases that the wave vector parallel or perpendicular to the magnetic field $\left(\vec{k} \, / / \, and \perp \vec{B}_0\right)$ to investigate the linear propagation of different electromagnetic waves. It is clear that the quantum effects increase or decrease the phase velocity of the modes depends on the external magnetic field. Besides, it has shown that the dispersion curves at $K = 0$ depend essentially on the positron-electron density ratio such as the positron density is increased to equal electron density, the wave frequency of the modes is increased..

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