

On the Fusion of Neutron/ Proton-Rich Colliding Nuclei Using Isospin-Dependent Potentials

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Abstract By using different isospin dependent proximity-type potentials, we performed a detailed study of neutron/proton-rich colliding nuclei with N/Z ratio between 0.5 and 2.0. Isotopes of three different series namely, Ne-Ne, Ca-Ca, and Zr-Zr are taken into account. A monotonous increase (decrease) in the fusion barrier positions (heights) using a unified second order nonlinear parametrization in the normalized fusion barrier positions and heights with $\left(\frac{N}{Z} - 1\right)$ is presented. These predictions are in good agreement with the available theoretical as well as experimental results. The fusion probabilities, however shows linear dependence. Further, the neutron/proton content plays dominant role at near barrier energies only. Our results are also independent of the model used as well as reaction partner and isospin content. More experiments are needed to verify our predictions.

Keywords: Heavy-ion fusion reaction, proximity potential, fusion barriers and cross sections.

1. INTRODUCTION

With the availability of radioactive-ion nuclear beams the fusion of colliding nuclei with the excess of neutron/proton ratio or near the drip line has attracted central position in the current days research [1-14]. The neutron-rich radioactive-ion beams have been applied to synthesize new, neutron-rich heavy nuclei [1]. This is because in the synthesis of heavy nuclei with n-rich projectile, one expect a higher survival probability of the completely fused system due to its lower fissility and lower excitation energies. One of the important use of radioactive-ion beam is to synthesize and study super heavy elements [2, 6].

Several experiential studies have also been undertaken in the literature to study the effect of varying the N/Z -ratio of the projectile and target nuclei upon the fusion cross sections [7-10, 15-21]. Recently, the enhancement of the fusion cross section has been observed for neutron-rich projectile [18-21]. This is due to the fact that neutron density distribution extended far beyond the range of normal nuclei leading to a larger barrier radius and hence to a lower height of the fusion barrier. Similarly, Bian *et al.* [7], employed the concept of static and dynamical Coulomb barrier to analyze the enhancement of the fusion cross section for neutron-rich systems using IQMD model. It has been suggested that the fusion yield would be further enhanced when the reaction is induced by unstable neutron-rich nuclei [22]. The neutron flow effect were also not proposed to understand the observed enhancement of the fusion cross sections [23, 24]. In addition, the fusion transfer at the neck region

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has been suggested [25, 26]. A microscopic description of the formation of neck in the fusion reaction remain a challenge to microscopic theories. The influence on sub-barrier fusion of processes such as, transfer [27] and breakup reaction [14, 28] is not yet clear; moreover, the effect of unusual structure, such as halos and skins [29], is being studied [8, 30]. Recently, Sun *et al.* [31], suggested that N/Z may be used as an experimental observable to extract neutron skin for neutron rich nuclei. All above experimental as well as theoretical information indicates that the dynamics of neutron/proton-rich nuclei is not fully understood and needs further attention.

The properties of various neutron-rich nuclei with different N/Z -ratio are studied in the literature e.g.: ${}^{9-10}_2\text{He}$ ($N/Z = 3.50-4.00$; where N and Z are the neutron and proton content of the nucleus), ${}^{6, 8, 9, 11}_3\text{Li}$ ($N/Z = 1.0, 1.67, 2.0, 2.67$), ${}^{22}_6\text{C}$ ($N/Z = 2.67$), ${}^{26-28}_8\text{O}$ ($N/Z = 2.25-2.50$), ${}^{31}_9\text{F}$ ($N/Z = 2.444$), ${}^{28, 32, 34}_{10}\text{Ne}$ ($N/Z = 1.8, 2.2, 2.40$), ${}^{30-32, 37}_{11}\text{Na}$ ($N/Z = 1.727-1.909, 2.364$), ${}^{40}_{12}\text{Mg}$ ($N/Z = 2.333$), ${}^{49-51}_{18}\text{Ar}$ ($N/Z = 1.722-1.833$), ${}^{60}_{20}\text{Ca}$ ($N/Z = 2.0$), ${}^{57-60}_{25}\text{Mn}$ ($N/Z = 1.28-1.4$), ${}^{68-78}_{28}\text{Ni}$ ($N/Z = 1.429-1.786$), ${}^{84, 86}_{30}\text{Zn}$ ($N/Z = 1.8, 1.87$), ${}^{90, 92}_{32}\text{Ge}$ ($N/Z = 1.813, 1.875$), ${}^{132}_{50}\text{Sn}$ ($N/Z = 1.64$), ${}^{123}_{47}\text{Ag}$ ($N/Z = 1.617$), ${}^{123-128}_{48}\text{Cd}$ ($N/Z = 1.563-1.667$) [32] and proton-rich are ${}^6_4\text{Be}$ ($N/Z = 0.50$), ${}^{10}_7\text{N}$ ($N/Z = 0.429$), ${}^{12}_8\text{O}$ ($N/Z = 0.50$), ${}^{17}_9\text{F}$ ($N/Z = 0.89$), ${}^{22}_{14}\text{Si}$ ($N/Z = 0.571$), ${}^{31}_{18}\text{Ar}$ ($N/Z = 0.722$), ${}^{34}_{20}\text{Ca}$ ($N/Z = 0.70$), ${}^{38, 39}_{22}\text{Ti}$ ($N/Z = 0.727, 0.773$), ${}^{45, 49}_{26}\text{Fe}$ ($N/Z = 0.731, 0.885$), ${}^{48, 49, 53}_{28}\text{Ni}$ ($N/Z = 0.714-0.75, 0.893$), ${}^{54}_{30}\text{Zn}$ ($N/Z = 0.80$), ${}^{217}_{92}\text{U}$ ($N/Z = 1.359$) etc. [33, 34].

A suitable set of models are therefore needed to study the dynamics of neutron/proton-rich colliding nuclei. A large number of theoretical models are available in the literature based upon the different assumptions [2-6]. Among them, proximity potential due to Blocki *et al.* [35], is well known for its simplicity and wider applications in different fields. Several modifications or refinements over the original proximity potential are also available in the recent time by including either up-to-date knowledge of the surface energy coefficient or nuclear radii [4, 36]. Various authors, modified or parametrized their approaches within the proximity concept [4]. All these modifications include new emerging degree of freedom i.e. isospin, either in radius formula, universal function and/or in surface energy coefficient [4]. Further, The outcome will definitely different if one use such type of models in the isospin plane.

Recently, one of us and collaborator, have carried out a detailed study involving large number of symmetric as well as asymmetric colliding nuclei using 16 proximity-type potentials [4]. Unfortunately, the maximum N/Z content of all the experimentally studied heavy-ion reactions is 1.60 (i.e. ${}^6\text{He} + {}^{238}\text{U}$) [8]. On the other hand, the first measurement with the proton drip line nucleus is of ${}^{17}\text{F} + {}^{282}\text{Pb}$ (with $N/Z = 1.473$) [33]. Therefore, a systematic study of nuclei having larger N/Z ratio using new proximity-type potentials is in demand. Further, it gives us a unique possibility to test the validity or accuracy of these models for the nuclei far away from the line of stability. Therefore, a systematic dependence of fusion barrier (heights and positions) and cross sections using various isospin dependent models on neutron excess is needed. Similar study was also presented by Puri *et al.* [3], where only Ca and Ni series were used. In the present

study, we extend the work to include new series like Ne-Ne (with N/Z ratio = 0.6-2.0) and Zr-Zr (with N/Z = 0.75-2.0) along with Ca-Ca (with N/Z = 0.5-2.0) series. The overall domain of N/Z ratio is from 0.5 to 2.0 for all series. The asymmetry parameter $A_s (= (N/Z) - 1)$ of the colliding nuclei varies between -0.5 and 1.0. Note that non zero value of A_s will involve complex interplay of the isospin degree of freedom which has strong role at intermediate energies as well [37]. Section 2, deals with fine points of the models in brief, Section 3 contains the results and summary is presented in Section 4.

2. FORMALISM

The total ion-ion interaction potential $V_T(r)$ comprises of nuclear and Coulomb part:

$$V_T(r) = V_N(r) + V_C(r). \quad (1)$$

Here $V_C(r) = (Z_1 Z_2 e^2 / r)$ is a good approximation, because fusion happens at a distance greater than touching configuration of the colliding pair.

The nuclear part of the ion-ion interaction potential $V_N(r)$ is calculated within the proximity concept. All proximity potentials are based upon the proximity force theorem, according to which [35], “*the force between two gently curved surfaces in close proximity is proportional to the interaction potential per unit area between the two flat surfaces*”. In original proximity potential [35], the nuclear part of the interaction potential $V_N(r)$ can be written as

$$V_N(r) = 4\pi\gamma b \bar{C} \Phi\left(\frac{r - C_1 - C_2}{b}\right) \text{ MeV}. \quad (2)$$

In this, \bar{C} is reduces radius with equivalent sharp radius R_i as

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \text{ fm} \quad (i = 1, 2). \quad (3)$$

Where $\Phi(\xi = \frac{r - C_1 - C_2}{b})$ is the universal function that depends on the separation between the surfaces of two colliding nuclei only. Both these factor do not depend on the isospin content. However, the last parameter γ , the surface energy coefficient, depends upon the neutron/proton excess as

$$\gamma = \gamma_0 \left[1 - k_s \left(\frac{N - Z}{A} \right)^2 \right], \quad (4)$$

where N, Z being the total number of neutrons and protons. In the original version, $\gamma_0 = 0.9517 \text{ MeV/fm}^2$ and $k_s = 1.7826$ [35]. Noted that for symmetric nuclear matter, $N = Z$, $\gamma = \gamma_0 = 0.9517 \text{ MeV/fm}^2$ indicating maximum strength of the potential. If we move to neutron (proton)-rich colliding nuclei with $N > Z$ ($N < Z$) then γ starts decreasing resulting in comparatively lesser attractive potential.

Later on, these coefficients were further improved by Möller and Nix with values $\gamma_0 = 1.2496 \text{ MeV/fm}^2$ and $k_s = 2.3$ [38]. This is labeled as Prox 88 [4, 39]. In the latest version of proximity potential [36], γ has form based on the precise neutron skin as

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$$\gamma = \frac{1}{4\pi r_0^2} \left[18.63(\text{MeV}) - Q \frac{(t_1^2 + t_2^2)}{2r_0^2} \right]. \quad (5)$$

The corresponding proximity potential is labeled as Prox 00 [4]. One of us and collaborator [4], modified above potential to include latest radius formula [40] and is denoted as Prox 00DP. Note that both Prox 00 and Prox 00DP has isospin dependent radius with slightly different constants whereas the factors surface energy coefficient γ and the universal function $\Phi(s)$ are same. In both newer versions of Bass (labeled as Bass 77 and Bass 80 in Ref. [4]) radius is slightly changed to

$$R_i = 1.16A_i^{1/3} - 1.39A_i^{-1/3} \text{ fm} \quad (i = 1, 2), \quad (6)$$

and then to sharp radius as is used in Prox 77 in the later version (i.e. Bass 80). Both newer versions of Winther (marked as BW 91 and AW 95 in Ref. [4]) has again similar expression for γ as is given in Prox 77 with slight difference that here isospin content is calculated separately for the target/ projectile. Whereas, first version due to Winther (labeled as CW 76 in Ref. [4]) does not have any γ dependence. Even radii are function of mass only. Both versions of Ngô (labeled as Ngô 75 and Ngô 80 in Ref. [4]) do not consider γ , but latest version of Ngô (Ngô 80) has isospin dependence in radius parameter. On the other hand, a complex isospin dependence in the universal function $\Phi(s)$ and radius is given in the version of Denisov [41]. Also by using the latest form of radius given in Ref. [40] in Denisov potential resulting in closer agreement with the experimental data for fusion barrier heights and cross-sections. This potential is labeled as Denisov DP [4]. All the above mentioned proximity-type potentials are able to reproduce the experimental fusion barriers within $\pm 10\%$ on the average [4].

In total, eight proximity-type potentials are used in the present study. Among them, three are basic proximity potentials (Prox 77, Prox 88, and Prox 00), Bass potential (Bass 80), Winther (AW 95) and Ngô (Ngô 80) each, and two newly modified potentials (Prox 00DP and Denisov DP) are used. The model due to Bass *et al.*, (Bass 80) used in the present analysis is independent of isospin dependence. For the detail of the models reader is refer to Ref. [4]. In most of the potentials and versions, modifications are made either through the surface energy coefficients or in nuclear radii. These two rather being technical parameters can have sizeable effects on the outcome of a reaction [5].

From these brief outlines, it is clear that much stress is made on the surface energy coefficients γ and nuclear radii to incorporate the isospin factor of a potential. Definitely, based on different assumptions and isospin dependence, different versions will respond to the collision of neutron-rich or -deficient nuclei differently compared to $N = Z$ nuclei.

3. RESULTS AND DISCUSSIONS

The present study deals with large variety of above mentioned potentials. Using these potentials, we firstly calculate the total ion-ion interaction potential using Eq. (1).

Once total ion-ion interaction potential is calculated, one can extract the barrier height V_B and barrier position R_B using conditions:

$$\frac{dV_T(r)}{dr}\Big|_{r=R_B} = 0; \text{ and } \frac{d^2V_T(r)}{dr^2}\Big|_{r=R_B} \leq 0. \quad (7)$$

Here we consider the collisions of three different series namely; ${}^A\text{Ca} + {}^A\text{Ca}$ (with $\frac{N}{Z} = 0.5$ to 2.0), and ${}^A\text{Zr} + {}^A\text{Zr}$ (with $\frac{N}{Z} = 0.75$ to 2.0) to cover wider mass range. We starts with the collision of $N = Z$ nuclei and then add (or remove) neutrons gradually from either of the colliding pairs till we reach $N = 2Z$ ($N = 0.5Z$) nuclei. In total, 150 such collisions involving different isotopes of different series are taken into account.

As a first step, we check the effect of addition or removal of neutrons on the nuclear part of the interaction potential $V_N(r)$ in different models. In Fig. 1, we display $V_N(r)$ as a function of internuclear distance r for the reactions of ${}^{70}\text{Zr} + {}^{70}\text{Zr}$, ${}^{70}\text{Zr} + {}^{76}\text{Zr}$, ${}^{76}\text{Zr} + {}^{80}\text{Zr}$, ${}^{80}\text{Zr} + {}^{80}\text{Zr}$, ${}^{80}\text{Zr} + {}^{86}\text{Zr}$, ${}^{86}\text{Zr} + {}^{90}\text{Zr}$, and ${}^{90}\text{Zr} + {}^{90}\text{Zr}$ using eight proximity-type potentials. Based upon the different assumptions used in different models, shape as well as strength of the potential differ accordingly. At the same time, the general shape at the surface region is same. In particular, Bass 80, Prox 00, and Prox 00DP have no repulsive core at shorter distances, whereas AW 95 follow Woods-Saxon-type form. On the other hand, Ngô 80, Denisov DP, Prox 77, and Prox 88 have repulsive core at shorter distances. In addition, four effects of addition (removal) of neutrons to $N = Z$ nuclei are clearly visible: (i) barrier height is decreased (ii) barrier position is increased (iii) depth of the potential is increased in all potentials except Denisov DP, and (iv) diffuseness of the potential is also changes. These effects will definitely influence the fusion probability at below barrier energies.

Therefore, before discussing the enhancement of the fusion cross section for the neutron-rich fusion reaction, we investigate the systematic dependence of fusion barriers on the neutron asymmetry parameter $A_s \left(= \frac{N}{Z} - 1 \right)$.

Using all above stated models, fusion barrier heights and positions are calculated for more than 150 colliding pairs. The variation in the fusion barrier positions and heights with neutron/proton content is analyzed in Figs. 2 and 3, respectively. Here, we display the variation of ΔR_B (%) and ΔV_B (%) defined as;

$$\Delta R_B \text{ (\%)} = \frac{R_B - R_B^0}{R_B^0} \times 100, \quad (8)$$

$$\Delta V_B \text{ (\%)} = \frac{V_B - V_B^0}{V_B^0} \times 100, \quad (9)$$

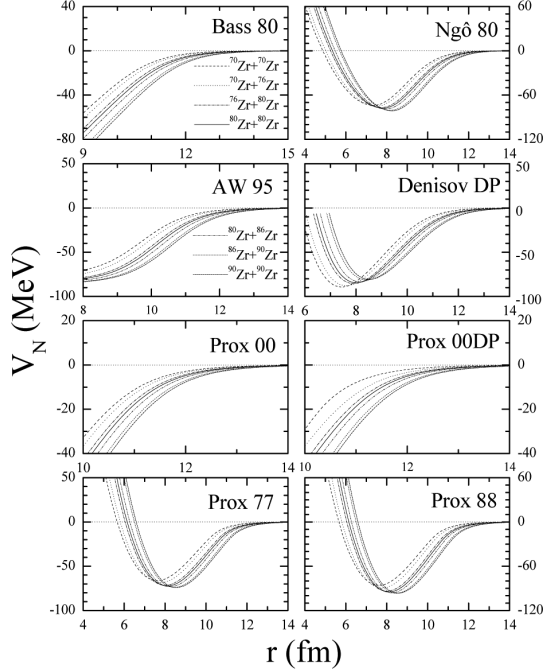


Figure 1: The nuclear part of the interaction potential V_N (MeV) as a function of internuclear distance r for the reactions of $^{70}\text{Zr} + ^{70}\text{Zr}$, $^{70}\text{Zr} + ^{76}\text{Zr}$, $^{76}\text{Zr} + ^{80}\text{Zr}$, $^{80}\text{Zr} + ^{80}\text{Zr}$, $^{80}\text{Zr} + ^{86}\text{Zr}$, $^{86}\text{Zr} + ^{90}\text{Zr}$, and $^{90}\text{Zr} + ^{90}\text{Zr}$ using eight sets of proximity-type potentials.

against asymmetry parameter $A_s \left(= \frac{N}{Z} - 1 \right)$ using eight set of potentials discussed above. Here, R_B^0 and V_B^0 are, respectively, the fusion barrier position and height corresponding to ($N = Z$) colliding nuclei and R_B and V_B refer for neutron/proton-rich colliding nuclei. The main advantage of these normalized variation is that it gives mass independent picture. For the present picture, we starts with the collision of $N = Z$ nuclei, then gradually add/remove neutrons from either of the colliding pair. For example, we started with the collision of $^{40}\text{Ca} + ^{40}\text{Ca}$, then add neutrons gradually, by keeping charges Z_1 and Z_2 always fixed. In this series, at the end of the chain we have the collision of $^{60}\text{Ca} + ^{60}\text{Ca}$. Similarly, if we remove the neutrons then we have at the end the collision of $^{30}\text{Ca} + ^{30}\text{Ca}$. It is clear from the figures, that all the models follow a unified non linear second order parametrization given as:

$$\Delta R_B (\%) = a \left(\frac{N}{Z} - 1 \right) + b \left(\frac{N}{Z} - 1 \right)^2, \quad (10)$$

$$\Delta V_B (\%) = c \left(\frac{N}{Z} - 1 \right) + d \left(\frac{N}{Z} - 1 \right)^2. \quad (11)$$

Here, a, b, c, and d are the constants varies from model to model and its values are displayed in Figs. 2 and 3. The above results are in good agreement with the work due to Puri *et al.*, for Ca and Ni series [3]. The available experimental as well as theoretical data is also displayed. All the available theoretical as well as experimental data follow our parametrization pattern very well except few points due to experimental uncertainty in different experimental setups. The theoretical data is taken from Refs. [42], whereas, the experimental data is taken from Refs. [43]. The above pattern indicates that, with the addition of neutron to $N = Z$ nuclei, fusion barrier position is increased and barrier height decreased, whereas, reverse happen for neutron -deficient nuclei. All models do not show much scattering from the middle curve as one move from $N = Z$ to very neutron-rich and -deficient colliding nuclei. Bass 80 and Ngô 80 show slight scattering for neutron-rich and -deficient nuclei. It may be due to the reason that the isospin dependence included in Ngô 80 model have not much effect on $\left(\frac{N}{Z}\right)$ ratio, whereas, Bass 80 is independent of such kind of dependence. We further note that the slope of the central lines also varies from model to model, whereas, the overall pattern is nicely explained by the above parametrizations. It may be due to the different assumptions used in different models.

Along with ΔR_B (%) and ΔV_B (%), we have also studied ΔV_N (%) and ΔV_C (%) using the same set of models and series (not shown here). We see that these variations show larger scattering due to the different assumptions resulting from different forms of radii, universal function, surface energy coefficients etc. The diffuseness parameter is also different in some potentials.

We further analyze the effect of addition or removal of neutrons on fusion probabilities. In Fig. 4, we display $\Delta\sigma_{\text{fus}}$ (%) with $A_s \left(\frac{N}{Z}-1\right)$ at energy $E = 1.02 V_B^0$. Where $\Delta\sigma_{\text{fus}}$ (%) is calculated similar to Eqs. (8) and (10) as

$$\Delta\sigma_{\text{fus}}(\%) = \frac{\sigma_{\text{fus}}(E_{\text{cm}}^0) - \sigma_{\text{fus}}^0(E_{\text{cm}}^0)}{\sigma_{\text{fus}}^0(E_{\text{cm}}^0)} \times 100, \quad (12)$$

with E_{cm}^0 and σ_{fus}^0 being the center-of-mass energy and fusion cross section for symmetric colliding pair. From this figure, it is clearly understood that all models follow linear dependence in fusion probabilities with A_s of the form.

$$\Delta\sigma_{\text{fus}}\% = \alpha \left(\frac{N}{Z} - 1 \right) \quad (13)$$

Here α is a constant and varies from model to model. Its values are displayed in the figure. The overall variation in its values is from 721.58 ± 40 to 1036.11 ± 150 . It indicates that some potentials strongly depend upon the N/Z content of the reaction. The shaded part represent the slight deviations from the linear dependence. As we noticed earlier, the slope of the linear curve is changes as one moves from one model to other. But the general behavior is nicely explained by linear parametrization [Eq. (13)]. It clearly indicates that our results are system as well as model independent. Also, the isospin degree of freedom introduced in different potentials does not affect

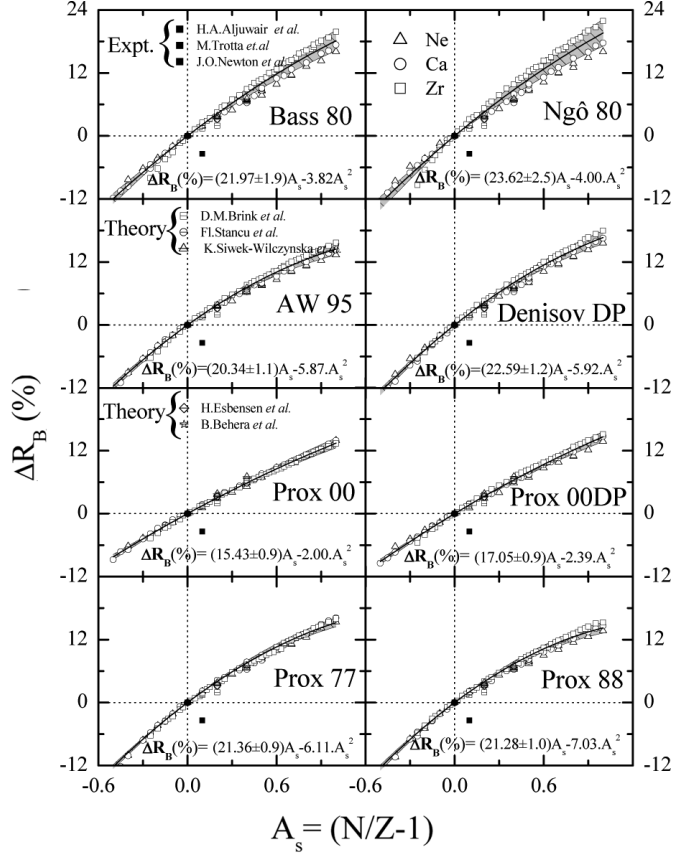


Figure 2: The normalized barrier positions ΔR_B (%) [Define in Eq. (8)] as a function of $A_s \left(= \frac{N}{Z} - 1 \right)$. We display the results of our calculations for the collisions of $^{A_1}\text{Ne} + ^{A_2}\text{Ne}$, $^{A_1}\text{Ca} + ^{A_2}\text{Ca}$, and $^{A_1}\text{Zr} + ^{A_2}\text{Zr}$ series using eight models along with other available theoretical and experimental values. The theoretical as well as experimental data reported here is taken from Refs. [42] and [43], respectively. The shaded areas denotes the deviation from the central solid lines.

our outcome. We carried out similar study at different energies i.e., $E = 1.075 V_B^0$, $1.125 V_B^0$, $1.25 V_B^0$, $1.375 V_B^0$, and $1.50 V_B^0$. We see that the effect at near barrier energy is more pronounced, whereas, at above barrier energies the effects are insignificant. We further test our parametrized results [Eqs. (10) and (11)] in Fig. 5 by using the formula due to Puri et al. [3] for neutron/proton-rich colliding pair as

$$\sigma_{\text{fus}}^{\text{fit}} (\text{mb}) = 10\pi R_B^0{}^2 (1 + \Delta R_B) \left(1 - \frac{V_B^0 (1 + \Delta V_B)}{E_{\text{cm}}} \right), \quad (14)$$

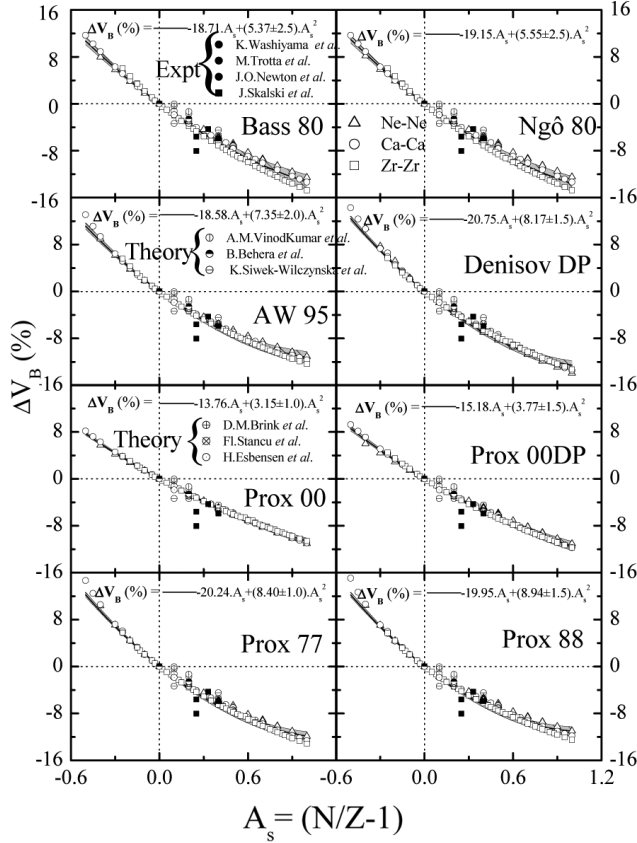


Figure 3: Same as Figure 2, but for ΔV_B (%) [Define in Eq. (9)].

for Ca-Ca series only. The corresponding results obtained using sharp cutoff formula

$$\sigma_{\text{fus}} (\text{mb}) = 10\pi R_B^2 \left(1 - \frac{V_B}{E_{\text{cm}}} \right), \quad (15)$$

are also displayed. From the figure, it is clear that our parametrized results are very close to the actual one obtained using Eq. (15) and is in agreement with earlier predictions [3]. This is the first time that such a wider mass range is taken into account and a complete isotopic dependence is studied using different proximity-type potentials. This study will certainly helpful to study various neutron/proton-rich nuclei lying between these limits.

4. SUMMARY

We analyzed the fusion of three different series namely, Ne, Ca, and Zr by covering the wider mass spectrum with $\left(\frac{N}{Z}\right)$ ratio between 0.5 and 2.0. We analyzed the

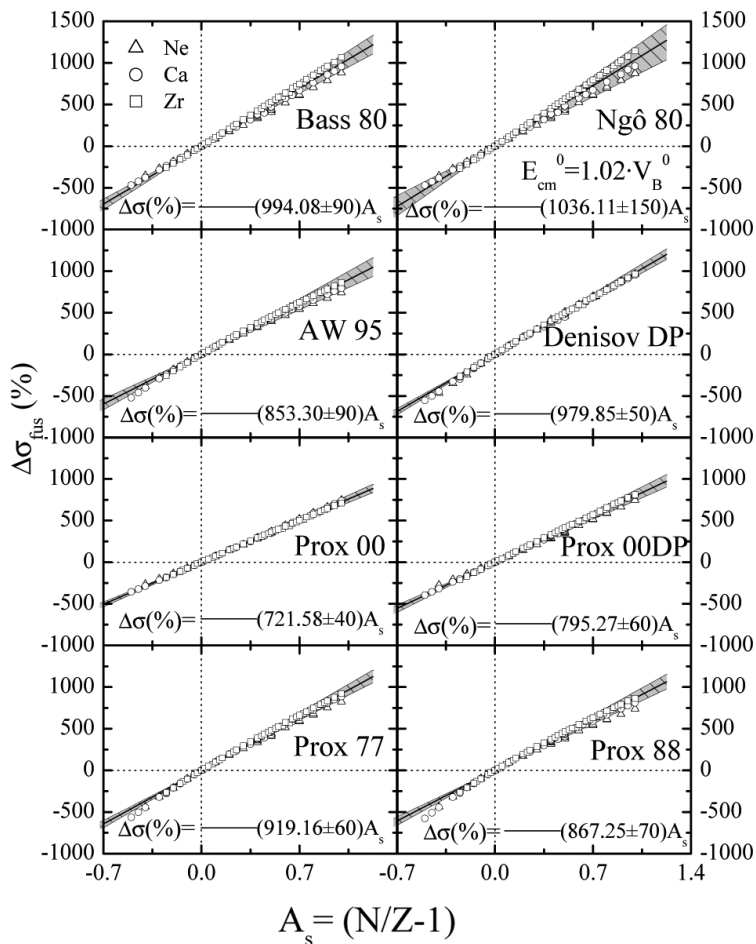


Figure 4: The normalized fusion probabilities $\Delta\sigma_{\text{fus}}$ (%) as a function of $A_s \left(= \frac{N}{Z} - 1 \right)$ for the collisions of $^{A_1}\text{Ne} + ^{A_2}\text{Ne}$, $^{A_1}\text{Ca} + ^{A_2}\text{Ca}$, and $^{A_1}\text{Zr} + ^{A_2}\text{Zr}$ series using eight sets of proximity-type model at incident energy $E_{\text{c.m.}} = 1.02 V_B$.

systematic dependence of fusion barriers on neutron excess and presented a unified second order non linear quadratic parametrization in fusion barrier heights and positions with $\left(\frac{N}{Z} - 1 \right)$ using eight isospin dependent proximity-type models for three different series. Our results are in good agreement with the available theoretical as well as experimental results. A linear dependence in the fusion probabilities is also presented. Further, the enhancement in fusion cross sections for neutron-rich nuclei due to lowering of fusion barrier heights is clearly seen, whereas, reverse happen for

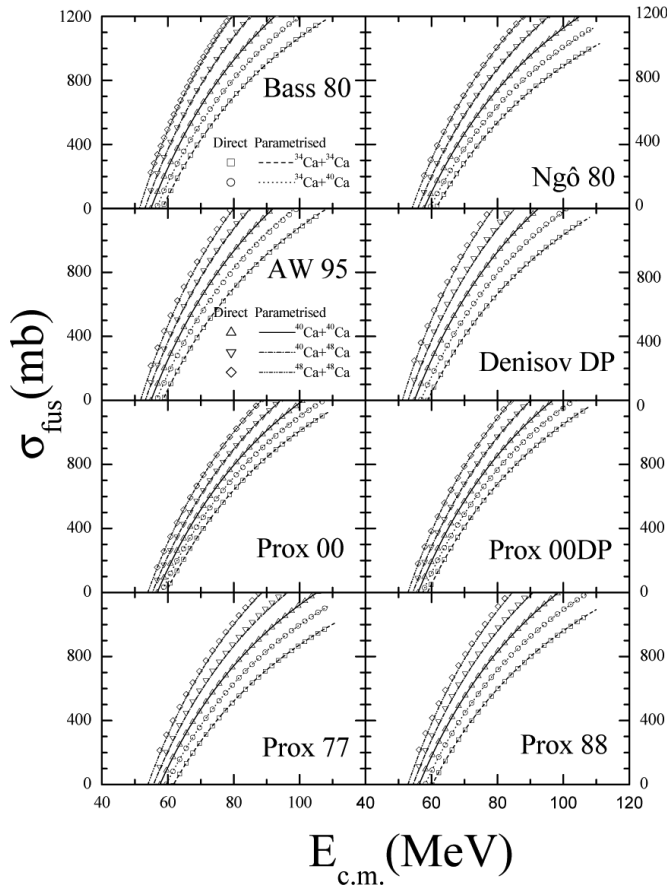


Figure 5: The parametrized fusion cross sections σ_{fus} (mb) (symbol) using [Eq. (14)] and the exact calculations (line) using [Eq. (15)] versus the center-of-mass energy $E_{\text{c.m.}}$ for the reactions of $^{34}\text{Ca} + ^{34}\text{Ca}$, $^{34}\text{Ca} + ^{40}\text{Ca}$, $^{40}\text{Ca} + ^{40}\text{Ca}$, $^{40}\text{Ca} + ^{48}\text{Ca}$, and $^{48}\text{Ca} + ^{48}\text{Ca}$ using eight sets of proximity-type potentials.

proton-rich nuclei. Along with this, our parametrization pattern is independent of the colliding nuclei as well as model and isospin content. At near barrier energies, N/Z content plays dominant role, whereas, the effect is insignificant at above barrier energies. More experiments are needed to verify our predictions.

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