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Fission in Rapidly Rotating Nuclei

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Abstract We study the effect of rotation in fission of the atomic nucleus ^{256}Fm using an independent-particle shell model with the mean field represented by a deformed Woods-Saxon potential and the shapes defined through the Cassinian oval parametrization. The variations of barrier height with increasing angular momentum, appearance of double hump in fission path are analysed. Our calculations explain the appearance of double hump in fission path of ^{256}Fm nucleus. The second minimum vanishes with increase in angular momentum which hints that the fission barrier disappears at large spin.

Keywords: Fission, Cassinian ovals, Rotating nuclei

1. INTRODUCTION

Nuclear fission is still an interesting topic for researchers because our understanding is incomplete due to the complexity of this process. Fission is a large amplitude collective motion of nucleons where nucleus evolves through different shapes. A proper understanding of fission process is very important in many areas of science and technology, for example, the understanding of nuclear fission helps us to improve our safety precautions of nuclear reactors. The term nuclear fission was introduced by Meitner and Frisch [1] to explain the experimental results as a division of a heavy nucleus into two lighter nuclei. Based on a liquid drop model in 1939, Bohr and Wheeler developed a theory of fission [2] as a competition between Coulomb energy and surface energy. Later, Hill and Wheeler [3] suggested that it is important to consider the microscopic nature of the nucleus by calculating the single-particle states of nucleons moving in a highly deformed nuclear potential to get a range of phenomena such as asymmetric fission, fission isomers etc. It was also realized that in such calculations the parameterization of shapes also plays a crucial role as the model should be capable of handling the binary shapes also. The nuclear shapes near the fission and beyond the fission point can be represented more efficiently by Cassinian ovals, which was introduced by Pashkevich [4]. The Cassinian ovals are a single-parameter family of curves convenient to approximate the shape of the nuclear surface while the nuclear volume is conserved. With this parameteriztion it is possible to the calculate fission barriers through an independent-particle shell model where the microscopic effects are taken care. In the present investigation, we look at the form of the fission barriers and potential energy surfaces in the nucleus ^{256}Fm obtained with independent-particle shell model with deformed Woods-Saxon potential. There are many works reported in recent literature [5, 6, 7, 8], explaining the process of fission. Most of them concern only the fission from non-rotating ^{256}Fm and in Ref. [9] the momenta of inertia are calculated. The present work focusses on

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the role of angular momentum, to which the barrier height for nuclear fission is quite sensitive.

2. THEORETICAL FRAMEWORK

The main development in this work is the extension of the Cassinian oval calculations to rotating nuclei. Here we present a brief outline of the shape parameterization [10] and the cranking method [11] for studying rotating nuclei. The shape of the nucleus in the zeroth-order approximation is taken to be the Cassinian ovaloid, the deviation being expanded in a series of Legendre polynomials. We can define new coordinates (R, x) in the plane containing the symmetry axis such that the coordinate line $R = const.$, is a Cassinian oval with the limits $0 \leq R < \infty$, and $-1 \leq x \leq 1$. The (R, x) coordinates relate to the cylindrical ones, (r, z) by the following expressions [4]:

$$R = [(z^2 + r^2)^2 - 2\varepsilon R_0^2(z^2 - r^2) + \varepsilon^2 R_0^4]^{1/4} \tag{1}$$

and

$$x = \frac{\mathrm{sign}(z)}{\sqrt{2}}\left\{1 + \frac{z^2 - r^2 - \varepsilon R_0^2}{[(z^2 + r^2)^2 - 2\varepsilon R_0^2(z^2 - r^2) + \varepsilon^2 R_0^4]^{1/2}}\right\}^{1/2}. \tag{2}$$

Here the deformation parameter ε varies from zero, for sphere to higher values which correspond to the separated fragments. The nuclear shape can be defined as a curve $R(x)$ that does not intersect any straight line $x = const.$, in more than one point. εR^2_0 can be understood as the distance between the origin of the coordinate and the focus of Cassinian ovals. The function $R(x)$ can be expand as

$$R(x) = R_0\left[1 + \sum_{m\geq 1}\beta_m Y_{m,o}(x)\right]. \tag{3}$$

The deformed Woods-Saxon potential with the spin-orbit interaction proportional to the potential gradient is used for the calculations [4],

$$V(r,z,\varepsilon,\hat{\beta}) = \frac{V_0}{1 + e^{\mathrm{dist}(r,z,\varepsilon,\beta)/a}} \tag{4}$$

Where dist(r,z,ε,$\hat{\beta}$) is the distance between the point r and the nuclear surface, a represents the surface diffuseness and V_0 stand for the depth of potential well. The single-particle energies with this potential are calculated by using the CASSINI code [10] which is most similar to work presented in Ref. [12] but for the shape parameterization. We consider only axially symmetric nuclear shapes.

A very direct way of investigating the properties of a rotating nucleus is to rotate (crank) it with certain angular velocity ω. The angle θ defines the tilt of the cranking axis ($\hat{n}$) with respect to the intrinsic z-axis within the principal xz-plane of the deformed potential, thus we get the cranking Hamiltonian as

$$H^{\omega} = H^{0} - \omega\hat{n}.\,\overline{J}, \tag{5}$$

where H^0 is the unperturbed (non-rotating) Hamiltonian and $\bar{J}$ is the angular momentum operator. A one dimensional cranking, also called a Principal Axis Cranking (PAC) can be performed by choosing the cranking axis by fixing θ. If we consider the rotation about the z-axis, we have

$$(\hat{n}.\overline{J})_{PAC_z} = J_z \text{ for } \theta = 0^{\circ}. \tag{6}$$

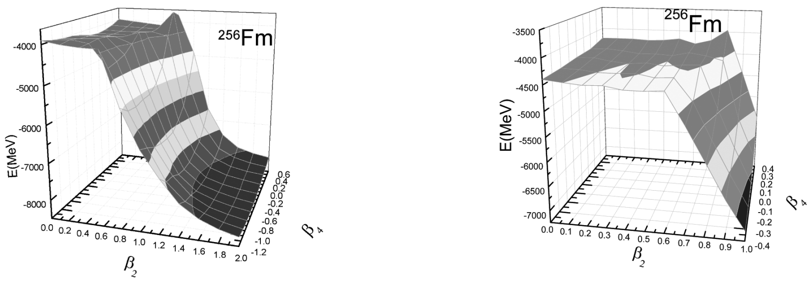
The expectation value of the above operator is diagonal and equal to the projection of single-particle angular momentum (Ω). Thus the angular frequency dependent terms are added to the Hamiltonian, to calculate the energy of the nucleus at a particular rotational frequency. A method of tuning is adopted such that the value of ω is tuned to achieve a desired angular momentum.

3. RESULTS AND DISCUSSION

The potential energy surface calculated at zero angular frequency ($\omega = 0$) which is eventually the zero angular momentum state ($I = 0\hbar$) for the considered even-even nucleus ^{256}Fm, is shown in Fig. 1(a). We can identify from Fig. 1(a) that the ground state corresponds to a lower deformation with a strong barrier towards the highly-deformed or binary shapes leading to fission. The fission path is mostly through the states with large negative β_4 values which correspond to the formation of the "neck" which at higher β_2 value, correspond to a binary shape.

For rotating nuclei, we find that the tuning to fixed spin plays a key role in obtaining the results at fixed angular momentum, which are more physical than those obtained at a constant ω. As explained in the formalism, spin tuning is the process of searching for the ω that leads to the desired angular momentum. Such a tuning has to be done at every point in the deformation mesh. By varying the angular momentum values from $0\hbar$ to higher values, the potential energy surfaces have been calculated and the results are shown in Fig. 1(b). We see that the increase in angular momentum leads to the development of second minimum (second hump) at larger deformations. Interestingly, at a high value of angular momentum of $100\hbar$, the first minimum vanishes but there is a local minimum at very high deformation with a large fission barrier.

The variations in the humps are vividly seen in the fission profiles shown in Fig. 2. From this figure, it can be observed that there is a sudden increase in the fission



(a) **(b)**

Figure 1: (a) Potential energy surface for ^{256}Fm at zero angular momentum. (b) Potential energy surface for ^{256}Fm at $I = 80\hbar$

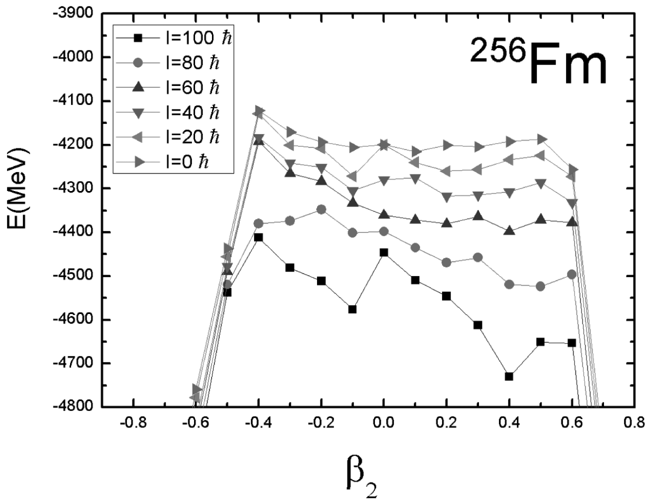


Figure 2: Fission profile for different values of spin (I) with negative and positive deformations

barrier from $I = 0\hbar$ to $I = 20\hbar$. As the angular momenta increases, the fission path develops a second hump. This can be seen as one moves from $I = 20\hbar$ to other higher values. At very high angular momenta, like at $I = 100\hbar$ one minimum vanishes. Nuclei at this region can sustain angular momenta up to a certain value which in case of ^{256} Fm is $\sim 65\hbar$ as suggested by the rotating liquid drop model (RLDM)[13]. The RLDM also suggests that the fission barrier should decrease as a function of angular momentum and hence beyond a critical angular momentum the fission barrier should vanish. Our calculations do not reveal such vanishing fission barriers. This discrepancy in the results could be attributed to the lack of considering the shell correction approach. Another drawback in our approach is that our cranking is restricted to rotation about symmetry axis (non-collective rotation). Fission is understood to be a collective phenomenon. Hence, one should consider rotation about an axis perpendicular to the symmetry axis i.e, either x or y axis. It is interesting to note that even without shell corrections, this model is able to explain the double humped fission barriers. This success of this model is attributed to the realistic potential considered with finite depth. Another possibility is that at lower angular momenta, a collective (liquid drop) behaviour mean that the nucleus would prefer to have an oblate shape which corresponds to negative values of β_2. Beyond sufficiently large angular momentum (close to the one which the nucleus can sustain without undergoing fission), there could be a transition from the oblate (non-collective) to prolate (collective) shape. Such shape transitions are predicted in gravitating rotating stars [14] and called as Jacobi transition. Though we do not have the configuration of collective prolate, we analyse how the oblate and prolate shapes compete within the non-collective rotation. From, Fig. 2, we can see that the barrier for fission through prolate shape is always smaller than that for the oblate shape.

At lower angular momenta (I=20$\hbar$) there is a strong preference towards an oblate shape. Such an oblate shape could not sustain higher angular momentum except for 100$\hbar$ which is rather an unphysical angular momentum for this nucleus.

4. CONCLUSIONS

Calculations with Cassinian oval paramaterization capable of explaining the binary shapes has been extended to study rotating nuclei. The cranking method is adopted along with the spin tuning to study the fission process at fixed angular momentum. The results obtained for the fission profiles and potential energy surfaces are analyzed in the case of ^{256}Fm. The vanishing second minima in the expected double humped fission for ^{256}Fm at higher angular momentum values was observed which would strengthen the fact that at higher angular momentum values fission barrier disappears. Our calculations do not explicitly reveal such vanishing fission barriers. This discrepancy could be attributed to the lack of considering the shell correction approach. We observed that the barrier for fission through prolate shape is always smaller than that for the oblate shape. A macroscopic-microscopic approach based on Strutinsky's method shall be adopted to improve the present approach. In spite of this, the present calculations could provide a better insight in to the structural and fission properties of ^{256}Fm nucleus.

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