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Effect of Magnetic Fields on Charged *B* Meson Decays

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ABSTRACT

The effect of magnetic fields on the leptonic decay of charged B meson $B^- \to l^- \bar{\nu}_l$ is investigated. The decay rate of the process is calculated both in the absence and presence of magnetic field. The non-perturbative parameters $f_{B^\pm}(\mathrm{B}), f^{'}_{B^\pm}$ and $M_{B^\pm}(\mathrm{B})$ are also estimated.

1. Introduction

The effect of magnetic field on the charged *B* meson decay is very important as well as interesting now-a-days. Magnetic field affects both neutral as well as charged *B* meson decay processes. We know very little about the charged *B* meson decay rates. Due to coupling of the magnetic field with the electric charge, there is splitting between the neutral and charged *B* mesons. Properties of mesons in strong magnetic field have been studied [1]. There is variation in the magnetic field produced from different sources and at different places. The strength of the magnetic field corresponds to, $\mathrm{eB} \approx 200 m_\pi^2$ during the cosmological electro-weak phase transition experienced by the early universe [2]. The magnetic field produced in heavy-ion collision experiments was of the order of $\mathrm{eB} \approx 0.1 m_\pi^2$ for SPS, $\mathrm{eB} \approx m_\pi^2$ for RHIC and $\mathrm{eB} \approx 15 m_\pi^2$ for LHC [3]. Charged and neutral vector meson under magnetic field has been studied in [4]. Weak decay of magnetized pions is studied [5]. Pion decay in magnetic fields is studied in [6]. There is effect of electromagnetic fields with Wilson fermions on the meson masses [7].

In this paper, we study the dependence of decay rate of charged *B* meson on the magnetic field. We have used Quantum Chromodynamics (QCD) and Lowest Landau Level (LLL) approximation for the charged *B* meson decay in the presence of magnetic fields. The leptonic decays of B mesons are studied [8] in the Standard Model (SM) and physics beyond the SM. We determine the non-perturbative parameters $f_{B^\pm}(\mathrm{B}),\ f^{'}_{B^\pm}$ and $M_{B^\pm}(\mathrm{B})$ in QCD which can be calculated using two lattice QCD simulation sets. The first set of lattice QCD simulations consist of quenched Wilson quarks and the second set of simulations is done by taking $N_f = 2+1$ flavors of dynamical staggered fermions [9, 10]. Analyzing the matrix elements H_μ and fitting correlators we extract the decay constants. Assuming *B* mesons can be observed using dynamical staggered [9], quenched Wilson [11, 12] and quenched overlap quarks [13], the mass of the *B* meson in the presence of the magnetic field is determined. The paper is structured as follows: in section 2, the decay rate is calculated using the Fermi's golden rule both in the absence and presence of magnetic field. In section 3, we discuss our results.

2. *B* Meson Decay Rate and Decay Constants

In the SM, the leptonic decay of charged *B* meson $B^-(p) \to l^-(k)\bar{\nu}_l(q)$ [8] originate from charged-current interactions due to W^- exchange between quark and lepton currents. Here, l^- is charged lepton $(l = e, \mu, \tau)$ and $\bar{\nu}_l$

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is the corresponding neutrino. The p, k and q denote the momenta to the B meson, the lepton and the antineutrino respectively. The effective Hamiltonian for this decay process can be written as [8, 13]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} \left[\bar{u}\gamma_\mu \left(1-\gamma^5\right) b\right] \times \left[\bar{l}\gamma^\mu \left(1-\gamma^5\right) v_1\right] + h.c., \tag{1}$$

The corresponding amplitude for the decay process

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} L^\mu H_\mu, \tag{2}$$

where G_F is Fermi's coupling constant, V_{ub} is the coupling constant for the coupling between b and u quark. The matrix elements of leptonic and hadronic contributions to the charged weak current are represented by L^μ and H_μ. The leptonic factor is defined by the matrix element,

$$L^\mu = \bar{u}_l(k)\gamma^\mu \left(1-\gamma^5\right) \mathrm{v}_\nu(q), \tag{3}$$

where u_l and v_ν represent bispin or solutions of the Dirac equation for the lepton and for the antineutrino, respectively. We can define matrix element for the hadronic factor

$$H_\mu = \left\langle 0 \middle| \bar{u}\gamma_\mu \left(1-\gamma^5\right) b \middle| B^-(p) \right\rangle. \tag{4}$$

The matrix element of the vector part of the weak current vanishes at zero magnetic field as it transforms as an axial vector [6]. The axial vector part of the weak current which is proportional to the B meson momentum $p_{\mu,}$ determines $H_{\mu}.f_{B^\pm}$ being the charged B meson decay constant defines the proportionality factor. The vector $F_{\mu\nu}p^\nu$ and the axial vector $-i\varepsilon_{\mu\nu\rho\sigma}F^{\nu\rho}p^\sigma/2$ forms the Lorentz structures for V-A decays in the presence of magnetic field where $F_{\mu\nu}$ is the Lorentz-tensor. With $F_{12} = -F_{21} = \mathrm{B}$, for the charged B meson at rest, the matrix element will be

$$H_\mu = e^{iM_{B^\pm}t}\left[f_{B^\pm} M_{B^\pm}\delta_{\mu 0} + if'_{B^\pm}\mathrm{eB}M_{B^\pm}\delta_{\mu 3}\right], \tag{5}$$

where $f'_{B^\pm}$ is the decay constant which emerge in the presence of magnetic field i.e. for $\mathrm{B} > 0$, and contribute to the second part of H_μ. The decay rate is calculated using the Fermi's golden rule

$$\Gamma = \int d\Phi \sum_{\{s\}} |\mathrm{M}|^2. \tag{6}$$

From equation (6), it is clear that in order to find the decay rate we have to integrate over the phase space Φ and sum over the intrinsic quantum numbers of the outgoing particles. In order to carry out the sum over the spins s_l and s_ν for $\mathrm{B}=0$, spin sums for the bispinors in equation (3) is needed. In case of the charged lepton, we have

$$\sum\nolimits_{s_l} u_l^{s_l}(k)\bar{u}_l^{s_l}(k) = \not{k} + m_l, \tag{7}$$

and for the neutrinos, we have

$$\sum\nolimits_{s_\nu} u_\nu^{s_\nu}(q)\bar{\mathrm{v}}_\nu^{s_\nu}(q) = \not{q} \tag{8}$$

where the tiny masses of the neutrinos have been neglected. In the absence of magnetic field, the decay rate is

$$\Gamma(\mathrm{B}=0) = \frac{G_F^2}{8\pi}|V_{ub}|^2 \left|f_{B^\pm}(0)\right|^2 \times \left[M_{B^\pm}^2(0) - m_l^2\right]^2 \frac{m_l^2}{M_{B^\pm}^3(0)}, \tag{9}$$

where, $f_{B^\pm}$ and $M_{B^\pm}$ are decay constant and mass of the charged B meson respectively at $\mathrm{B}=0. m_l$ denotes the lepton mass.

The dependence of Dirac equation on the magnetic field affects the bispin or solution u_l. The quantized solutions correspond to Landau levels [14] are states of definite angular momentum in the z direction. A degeneracy proportional to the flux $\mathrm{eB}.L^2$ of the magnetic field through the area L^2 of the system is carried out by each Landau level. Corresponding to the n^{th} Landau level, the energies of the states are bounded from below by $\sqrt{m_l^2 + 2n\mathrm{eB}}$. In case of strong magnetic fields, considering $\mathrm{eB} \gg m_l^2, n=0$ state will only contribute. This will restrict leptons in the lowest Landau level (LLL) neglecting levels with $n>0$. Since the s_l spin is antiparallel to the magnetic field, the LLL states allow only one-dimensional motion aligned with the magnetic field. Hence there will be equivalent of equation (7)

$$\sum_{LLL} u_l^{s_l}(k)\bar{u}_l^{s_l}(k) = \left(\not{k}_\parallel + m_l\right).\frac{1-\sigma^{12}}{2} \tag{10}$$

where, $\not{k}_\parallel = k^0\gamma^0 - k^3\gamma^3$ is emerged for the one-dimensional nature of the LLL states. The second factor of equation (10) involves the relativistic spin operator $\sigma^{12} = i\gamma^1\gamma^2$ by which the negative spin states are projected. The sum over the LLL-degeneracy contributes to the third factor. The magnetic field does not affect the spin sum for the neutrino. In equation (10), we use the spin sum in order to calculate the decay rate and obtain,

$$\Gamma(\mathrm{B}) = \mathrm{eB}\frac{G_F^2}{2\pi}|V_{ub}|^2 \times \left|f_{B^\pm}(\mathrm{B}) + if'_{B^\pm}(\mathrm{B})e\mathrm{B}\right|^2 \frac{m_l^2}{M_{B^\pm}(\mathrm{B})}. \tag{11}$$

In equation (11), we assume that magnetic field does not affect $G_F V_{ub}$ due to its low energy scale which is much smaller than the mediating particle of the weak interaction boson. The impact of the magnetic field on the decay rate can be quantified by the ratio

$$\frac{\Gamma(\mathrm{B})}{\Gamma(0)} = 4\frac{\left|f_{B^\pm}(\mathrm{B})\right|^2 + \left|f'_{B^\pm}(\mathrm{B})\mathrm{eB}\right|^2}{\left|f_{B^\pm}(0)\right|^2}. \times\left(1-\frac{m_l^2}{M_{B^\pm}^2(0)}\right)^{-2}\frac{\mathrm{eB}}{M_{B^\pm}(0)}\frac{1}{M_{B^\pm}(\mathrm{B})}, \tag{12}$$

where the constants G_F and V_{ub} cancel. The result obtained in equation (12) is valid only for the limit $\mathrm{eB} \gg m_l^2$.

3. Results and Discussions

In order to find out $\Gamma(\mathrm{B})/\Gamma(0)$, of equation (12), there is need of determining the non-perturbative parameters $f_{B^\pm}(\mathrm{B}), f'_{B^\pm}$ and $M_{B^\pm}(\mathrm{B})$ in QCD which can be calculated using two lattice QCD simulation sets. The first set of lattice QCD simulations consist of quenched Wilson quarks. In this set, the lattice spacing lie in the range $0.047\mathrm{fm} \le a \le 0.124\mathrm{fm}$. We set $M_{B^\pm}(0) \approx 16\mathrm{GeV}$. The second set of simulations is done by taking $N_f = 2+1$ flavors of dynamical staggered fermions [9, 10]. In this set, the lattice spacing spans over $0.1\mathrm{fm} \le a \le 0.22\mathrm{fm}$. We set $M_{B^\pm}(0) \approx 5.3\mathrm{GeV}$. Analyzing the matrix elements given in equation (5) and fitting three correlators [5, 7] we have

$$C_{OP}(t) = C_{OP}\left[e^{-M_{B^\pm}t} \pm e^{-M_{B^\pm}(N_t - t)}\right], \quad O = P, A, V \tag{13}$$

where, $P = \bar{u}\gamma^5 d, A = \bar{u}\gamma_0\gamma^5 d, V = \bar{u}\gamma_3 d$. C_{PP} takes positive sign, C_{AP} and C_{VP} take negative sign due to time reversal properties of the correlators. We extract the decay constants via

$$f_{B^\pm} = Z_A.\frac{\sqrt{2}c_{AP}}{\sqrt{M_{B^\pm}c_{PP}}}, \quad if'_{B^\pm}eB = Z_V.\frac{\sqrt{2}c_{VP}}{\sqrt{M_{B^\pm}c_{PP}}}, \tag{14}$$

Z_A and Z_V being the multiplicative renormalization constants of the axial vector and vector currents.

For $\mathrm{B} > 0$, correlation functions, C_{VP} vanishes at B = 0 [5]. The mass of the B meson in the presence of the magnetic field can be written as

$$M_{B^\pm}(\mathrm{B}) = \sqrt{M_{B^\pm}^2(0) + \mathrm{eB}}, \tag{15}$$

Assuming B mesons to be point like free scalar particles which can be observed using dynamical staggered [9], quenched Wilson [11, 12] and quenched overlap quarks [13]. Here, we consider $B \to \tau\bar{\nu}_\tau$ decay process. The decay rate formula given by equation (11) is valid for magnetic fields above squared lepton masses. From Figures 1, it is clear that both f_B and $f'_B\mathrm{eB}$ increase with magnetic field B. In Figure 2, we have shown the dependence of decay rates on the magnetic fields using both staggered fermions and Wilson quarks. The results reveal that there is significant enhancement of Γ with magnetic field Therefore, the average lifetime of the charged B meson decreases. Similarly, for $B \to \mu\bar{\nu}_\mu$ decay process, we will get similar type of results.

We use the formula, $f_B/f_B(0) = [1 + c_1|\mathrm{eB}|]M_B(0)/M_B$ & $f'_B/f_B(0) = d_0 M_B(0)/M_B$. Taking the value $c_1 = 1$ & $d_0 = 1$, we plot the graph of $f'_B\mathrm{eB}/f_B(0)$ & $f_B/f_B(0)$ with respect to eB using staggered fermions. When the magnetic field is high, due to uncertainties of data we have to include systematic error. The expression for decay constant at high can be changed to $f'_B/f_B(0) = \left[d_0 + d_1|\mathrm{eB}| + d_2|\mathrm{eB}|^2\right]M_B(0)/M_B$. We have to use non-zero values of d_1, d_2 if we have to perform lattice QCD simulations using quenched Wilson quarks.

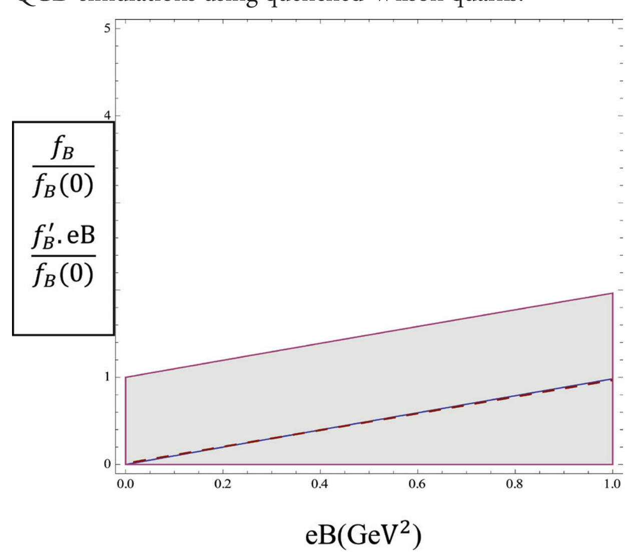

Figure 1: Variation of $f'_B\mathrm{eB}/f_B(0)$ & $f_B/f_B(0)$ with respect to eB using staggered fermions. The lower triangular portion represents the variation of $f'_B\mathrm{eB}/f_B(0)$ and the whole trapezium portion (including triangular portion) represents the variation of $f_B/f_B(0)$ with respect to eB(GeV^2).

According to equation (12), the ratio of muonic and tau leptonic decay rate is independent of the magnetic field, $\mathrm{eB} \gg m_\tau^2$. The mass of charged B mesons and that of muons as well as tau leptons have been taken from the Particle Data Group [15] $\mathrm{M}_{B^\pm}(0) = 5279.32 \pm 0.14\mathrm{MeV}, m_\mu =$

$105.6583745 \pm 0.0000024$ MeV and $m_\tau = 1776.86 \pm 0.12$ MeV.

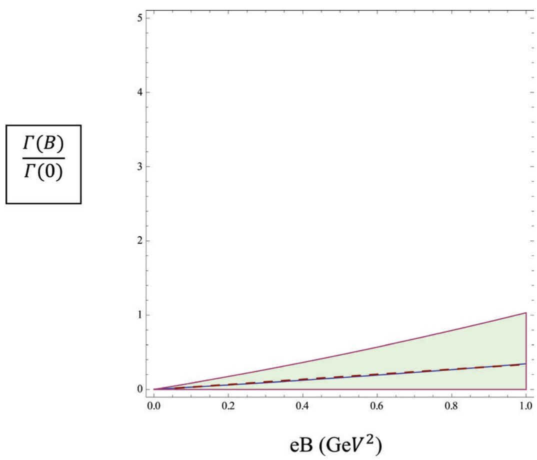

Figure 2: Variation of $\Gamma(\mathrm{B})/\Gamma(0)$ with respect to eB using staggered fermions and Wilson quarks. The lower triangle represents the staggered fermions case and the big triangle (including small triangle) represents the Wilson quarks case.

In the absence of magnetic fields,

$$\frac{\Gamma(\mathrm{B})_{B\to\mu\bar{\nu}_\mu}}{\Gamma(\mathrm{B})_{B\to\tau\bar{\nu}_\tau}} = \frac{m_\mu^2}{m_\tau^2} = (3.536 \pm 0.48)\times 10^{-3}. \qquad (16)$$

$$\frac{\Gamma(0)_{B\to\mu\bar{\nu}_\mu}}{\Gamma(0)_{B\to\tau\bar{\nu}_\tau}} = (4.493 \pm 0.002)\times 10^{-3}. \qquad (17)$$

Comparing equations (16) and (17), the ratio of decay rates of the decays $B\to\mu\bar{\nu}_\mu$ and $B\to\tau\bar{\nu}_\tau$ in the presence of magnetic fields is reduced by about 20% than that in the absence of magnetic fields. Finally we have reached the point that, in the presence of magnetic fields, the tau leptonic decay becomes more dominant than muonic decay at B.

Conclusion

We have come to the conclusion that in the presence of magnetic fields, the decay rate of charged B meson $B^- \to l^-\bar{\nu}_l$ depends on decay constant $f'_{B^\pm}$ in addition to the ordinary B meson decay constant $f_{B^\pm}$. The decay rate increases in the presence of magnetic field which is reflected in Figure 2.

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