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Appearance / Disappearance of Magic Number in Light Nuclei

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ABSTRACT

The shell structure of a nucleus is important to study their observed characteristic features. The classic magic numbers are successful in explaining the nuclear properties for nuclei lying near the stability line. The advent of radioactive ion beam facilities has permitted to examine nuclei in their extreme proton to neutron ratio. The light exotic nuclei were found to exhibit unique shell closure behaviour which is different from the medium mass nuclei near the stability line. The two nucleon separation energy difference systematics was used as a probe to study the magic character of light nuclei. New proton and neutron magic numbers were predicted among the available even Z isotopes and even N isotones. For certain systems, the classic magic numbers were found to be non−magic, while for some systems the magic property is retained even at the drip lines. The shell closure behaviour predicted is found to depend on the version of the mass table.

1. Introduction

The nuclei with nucleon numbers 2, 8, 20, 28, 50, 82 and 126 are found to be extremely stable compared to neighbouring nuclei. The specific nucleon numbers are referred to as “magic number” and they remain magic for nuclei along the stability line. It was Mayer and Haxel [1, 2] who put forward an explanation for the extra stability possessed by nuclei with magic nucleon numbers. The nuclear shell model developed in accordance with atomic shell theory was successful in accounting for the observed magic number. According to the theory, the nucleons are arranged in well-defined nucleonic single-particle levels. A large gap between single-particle levels is noted as a shell gap and it marks the signature for a magic nucleon number. The shell model works well for nuclei near the stability line. The scenario changes while moving towards neutron/proton drip-line nuclei where there is an extreme neutron to proton ratio. With the advent of technology, the study on shell closure and magicity in light exotic nuclei has gained much interest. A number of experimental observations have supported the appearance of the new magic number [3, 4, 5] and disappearance of the classic magic number [6, 7] in light exotic nuclei. Similarly, the experimental evidence [8] in support of mixing of $1s_{1/2}$ and $0p_{1/2}$ levels exclude the $N = 8$ magic number in ^{11}Li. These experimental observations recommend that the shell gaps are not universal [9] but depend on the relative balance between neutron and proton numbers. Different theoretical studies were performed in connection with identifying magic numbers in light exotic nuclei [10, 11]. Various theoretical methods are available to determine the shell closure in nuclei. One among them is the potential energy surface analysis using cluster core model [12, 13] which identified $N = 6$ and 14 or 16 and $Z = 6$ and 14 as the possible magic number for nuclei near the proton drip-line. Another important method is the separation energy systematics which can be used as a probe to study the magic number. Similar to atomic theory, the energy needed to remove the last nucleon from an atomic nucleus varies with nucleon number. The nuclei with filled shells are more tightly bound than the neighbouring nuclei and hence the separation energy will be relatively higher. Thus the nucleon separation energy systematics exhibits discontinuities when plotted as a function of nucleon number and this forms the basis of the present work.

2. Methodology

The appearance and disappearance of nucleon shell closure among light nuclei are studied from the two nucleon separation energy difference systematics. Nucleon separation energy is determined as the difference in the binding energy of the nucleus whose nucleon separation energy has to be evaluated and the nucleus which is a deficit of two nucleons.

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The $2n$ and $2p$ separation energy are mathematically written as follows,

$$S_{2n}(A,Z) = BE(A,Z) - BE(A-2,Z), \text{ and} \quad (1)$$

$$S_{2p}(A,Z) = BE(A,Z) - BE(A-2,Z-2), \quad (2)$$

where $BE(A, Z)$ is the binding energy of a nucleus with the mass number A and charge number Z. The binding energy $BE(A, Z)$ is evaluated using the liquid drop model (LDM) expression which is a modified form of Bethe-Weizsäcker formula proposed by Samanta and Adhikari [14]. The formula was suggested as an extension of Bethe-Weizsäcker form such that it is applicable for light nuclei from Li onwards. The modified binding energy expression proposed by Samanta and Adhikari takes the following form:

$$BE(A,Z) = a_V A - a_S A^{\frac{2}{3}} - a_C Z(Z-1) A^{-\frac{1}{3}} - E_a^{new} + \delta^{new}, \quad (3)$$

where the terms in their order are volume, surface, Coulomb, asymmetry and pairing energy respectively. The superscript *new* in the last two terms is to indicate that the asymmetry energy E_a and the pairing energy δ in their original form were modified to the new form. The relation between E_a^{new}, E_a and δ^{new}, δ are presented below,

$$E_a^{new} = \frac{E_a}{1+e^{-A/k}} \text{ with } E_a = \frac{a_A (A-2Z)^2}{A}, \text{ and} \quad (4)$$

$$\delta^{new} = \left(1 - e^{-A/C}\right)\delta \text{ with } \delta = \begin{cases} +a_p A^{-1/2} \ \text{even } N - \text{even } Z \\ -a_p A^{-1/2} \ \text{odd } N - \text{odd } Z \\ 0 \ \text{odd } A \end{cases} \quad (5)$$

The constants used in the above equation has the values a_V = 15.85 MeV, a_C = 0.71 MeV, a_S = 18.34 MeV, a_A = 23.21 MeV, a_p = 12 MeV, k = 17 and c = 30. The asymmetry energy and pairing energy were modified by an exponential factor as given in Eqs. (4) and (5). Both the terms E_a^{new} and δ^{new} approaches to E_a and δ with increase in mass number A. This is because the additional exponential factors introduced in defining E_a^{new} and δ^{new} becomes unity with the increase in mass number. As a representative case, Fig. 1 shows the variation of E_a^{new} and δ^{new} with respect to E_a and δ for $N = 22$ isotones for even proton number. From the figure, the difference between E_a^{new} and E_a and similarly those between δ^{new} and δ decreases with an increase in proton number. Thus the modification in the asymmetry and pairing energy is significant for light mass nuclei.

The separation energy systematics exhibits a sudden drop in separation energy at the so-called nucleon magic numbers. Thus the region exhibiting the discontinuity in separation energy can be taken as an indication of filled shell closure. Rather than the separation energy systematics, the study of the difference between calculated and experimental separation energy is more appropriate to analyse the appearance of new magic number and disappearance of classic magic numbers [15]. So far, most of the experimental works concerning the nuclear shell closure were focussed mainly on light exotic nuclei. Since the modified Bethe-Weizsäcker formula could account for the binding of light nuclei well compared to unmodified formula, in the present work we consider only light nuclei with $8 \le Z(N) \le 22$ and the respective available isotopes and isotones. The two nucleon separation energy difference is taken as $\Delta S_i = (S_i)_{Expt} - (S_i)_{LDM}$ where $i = 2n, 2p$. The experimental two nucleon separation energy $(S_i)_{Expt}$ was retrieved from AME2016 [16] table and $(S_i)_{LDM}$ was evaluated using LDM [14] expression which is the modified Bethe-Weizsäcker formula. A sudden decrease in the $\Delta S_{2n(2p)}$ value at $N(Z) + 2$ indicates the closed-shell structure property of the nuclei at $N(Z)$.

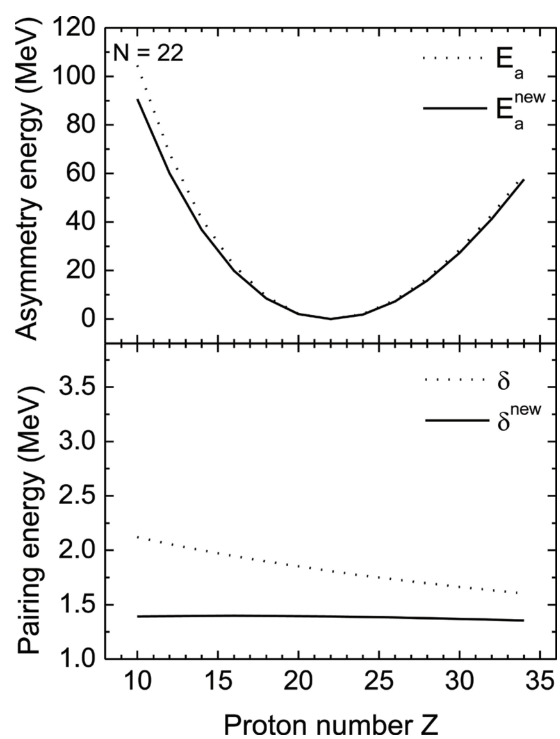

Figure 1: The variation of asymmetry (top) and pairing energy (bottom) in their modified form E_a^{new} and δ^{new} (solid) relative to their original form E_a and δ (dotted) for $N = 22$ isotones for even proton number Z.

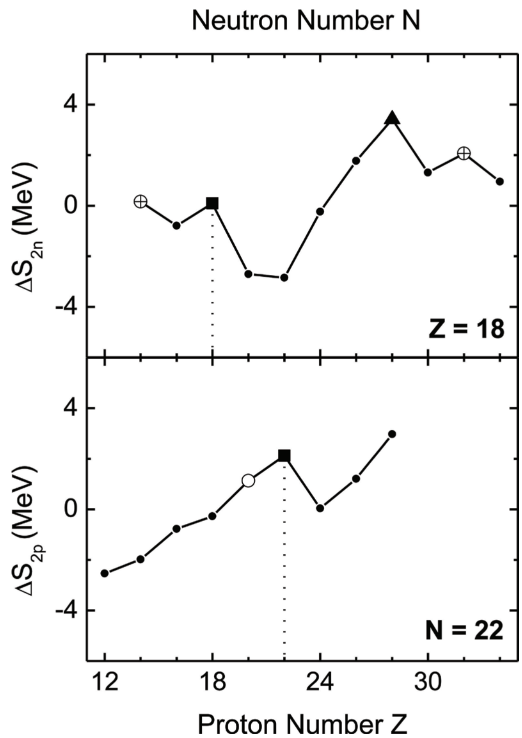

Figure 2: (top) The ΔS_{2n} systematics for Z = 18 isotopes; (bottom) ΔS_{2p} systematics for N = 22 isotones. The solid square with dotted vertical line represents $N = Z$ cases, open circle represents the disappearance of classic magic number, a circle with + center represents the appearance of new magic number and a solid up triangle represents the existence of classic magic number.

3. Results

In the present calculation, ΔS_i systematics for even Z isotopes with $8 \leq Z \leq 22$ and for even N isotones with $8 \leq N \leq 22$ were studied. Since a complete shell closure occurs at even nucleon numbers, the ΔS_i systematics and the search for the magic character was limited to even nucleon numbers. Figure 2 presents the ΔS_{2n} systematics for Z = 18 isotopes as a function of neutron number in the top panel and ΔS_{2p} variation for N = 22 isotones as a function of proton number in the bottom panel. A sudden decrease in ΔS_i was taken as an indication of the shell closure and hence the presence of a magic number. In Fig. 2, the solid square with dotted vertical line indicates $N = Z$ systems where there is a decrease in ΔS_i at $N = Z$ =18 in the top panel and at $Z = N = 22$ in the bottom panel. The open circle at Z = 20 among N = 22 isotones in the bottom panel of Fig. 2 marks the disappearance of classic shell closure where there is an unexpected increase in ΔS_{2p} value. However, the classic shell closure at N = 28 is retained among Z = 18 isotopes indicated by a solid up triangle in the top panel of Figure 2. In addition to classic shell closures, the nucleon numbers where there is a drop in ΔS_i value are considered as new magic numbers and are denoted by a circle with + center. As a representative case, the emergence of two new magic numbers at N = 14, 32 among Z = 18 isotopes can be observed in the top panel of Figure 2. The variation in ΔS_i value where $i = 2n$, $2p$ relative to the neighbouring nuclei is calculated as $\Delta^2 S_i(j) = \Delta S_i(j+2) - \Delta S_i(j)$ where $j = N$ for $i = 2n$ and $j = Z$ for $i = 2p$. In all the cases studied, the presence or absence of shell closure is reported only when there is a significant change in ΔS_i value such that $|\Delta^2 S_i| \geq 0.5$ MeV. The cases with $|\Delta^2 S_i| < 0.5$ MeV are neither marked nor mentioned in the text. Since $|\Delta S_i| \leq$ 6 MeV for all the cases considered, it is clear that the modified formula of Samanta and Adhikari can predict the experimental binding energy of light nuclei.

Tables 1 and 2 present the $\Delta^2 S_i$ values for certain even Z isotopes and even N isotones with a remark on whether the magic character is either preserved/destroyed or new magicity appears. A negative $\Delta^2 S_i$ at specific nucleon number refers to the shell closure property or extra stability for the system where there is a drop in ΔS_i value relative to its neighbouring nuclei. In some cases, negative $\Delta^2 S_i$ value is observed at the classic magic number indicating the existence of classic magic number. However, in certain cases, negative $\Delta^2 S_i$ occurs at nucleon numbers other than classic magic numbers which mark the appearance of new magic numbers. In certain classic magic nucleon numbers, an unexpected increase in ΔS_i or a positive $\Delta^2 S_i$ is observed thus indicating the disappearance of classic magic number.

Table 1: The $\Delta^2 S_{2n}$ values for available isotopes of even Z nuclei which exhibits a notable shell closure.

Z	N	$\Delta^2 S_{2n}$	Remarks
8	8	−8.548	N = Z = 8
	16	−3.482	N = 16, Appearance of new magic number

10	10	−4.425	N = Z = 10
	20	1.148	N = 20, Disappearance of classic magic number
12	12	−5.137	N = Z = 12
	16	−0.585	N = 16, Appearance of new magic number
	20	2.129	N = 20, Disappearance of classic magic number
14	14	−5.927	N = Z = 14
	28	1.083	N = 28, Disappearance of classic magic number
16	16	−3.058	N = Z = 16
	20	−0.747	N = 20, No change in classic magic number
	26	−0.642	N = 26, Appearance of new magic number
	28	1.184	N = 28, Disappearance of classic magic number
	30	−0.841	N = 30, Appearance of new magic number
18	14	−0.951	N = 14, Appearance of new magic number
	18	−2.803	N = Z = 18
	28	−2.105	N = 28, No change in classic magic number
	32	−1.116	N = 32, Appearance of new magic number
20	20	−4.872	N = Z = 20
	28	−3.061	N = 28, No change in classic magic number
	32	−1.642	N = 32, Appearance of new magic number
22	20	0.576	N = 20, Disappearance of classic magic number
	22	−1.953	N = Z = 22
	28	−2.083	N = 28, No change in classic magic number

From Table 1, other than $N = Z$ systems, existence of classic magicity at $N = 20$ is retained for sulphur, $N = 28$ at argon, calcium and titanium isotopes where $\Delta^2 S_{2n} < -0.5$ MeV. On the other hand the disappearance of classic magicity at $N = 20$ among the isotopes of neon, magnesium, titanium and at $N = 28$ among the isotopes of silicon, sulfur where a positive $\Delta^2 S_i$ is noted in Table 1. New magic numbers at $N = 14$ among argon, $N = 16$ among oxygen, magnesium, $N = 26$, 30 among sulphur, $N = 32$ among argon, calcium isotopes were observed with a negative $\Delta^2 S_{2n}$ in Table 1. Similar to Table 1, Table 2 presents the appearance of new magicity, disappearance and retention of classic magicity among proton number by studying ΔS_{2p} systematics for light even N isotones. The classic proton magic number at $Z = 8$ is retained at the extreme proton deficient isotones of $N = 14$ and 16 which possess a large negative $\Delta^2 S_{2p}$ value as noted in Table 2 while it disappears among $N = 10$ and 12 isotones with a positive $\Delta^2 S_{2p}$. Similarly, the classic proton shell closure at $Z = 20$ disappears for $N = 22$ isotone with a positive $\Delta^2 S_{2p}$ value even though the respective nuclei lie near the stability line. New shell closure at $Z = 6$ for $N = 8$, at $Z = 14$ for $N = 16$, 18, 20 isotones is predicted by the separation energy difference systematics with $\Delta^2 S_{2p} < -0.5$ MeV in Table 2.

Table 2: The $\Delta^2 S_{2p}$ values for available isotones of even N nuclei which exhibits a notable shell closure behaviour.

N	Z	$\Delta^2 S_{2p}$	Remarks
8	6	−1.451	Z = 6, Appearance of new magic number
	8	−8.109	Z = N = 8
10	8	2.130	Z = 8, Disappearance of classic magic number
	10	−4.519	Z = N = 10
12	8	1.433	Z = 8, Disappearance of classic magic number
	12	−5.245	Z = N = 12
14	8	−2.034	Z = 8, No change in classic magic number
	14	−5.957	Z = N = 14
16	8	−6.035	Z = 8, No change in classic magic number
	14	−0.805	Z = 14, Appearance of new magic number
	16	−3.028	Z = N = 16
18	14	−2.100	Z = 14, Appearance of new magic number
	18	−2.743	Z = N = 18
20	14	−1.013	Z = 14, Appearance of new magic number
	20	−4.537	Z = N = 20
22	20	0.992	Z = 20, Disappearance of classic magic number
	22	−2.082	Z = N = 22

In most of the cases, the separation energy values recorded in mass tables for nuclei lying either at neutron drip-line or proton-drip line are non-experimental or estimated values. Since the present work relies on separation energy difference systematics which depends on the experimental separation energy values, an update in the experimental mass table will affect the calculation. A schematic representation of ΔS_i systematics for $Z = 18$ isotopes and $N = 12$ isotones with experimental separation energy values taken from three different mass tables namely AME2003 [17], AME2012 [18] and AME2016 [16] are presented in Fig. 3.

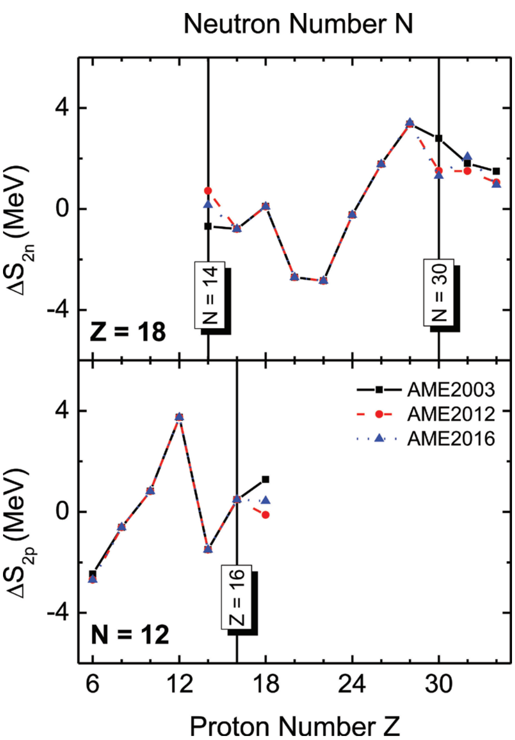

Figure 3: Separation energy difference systematics: (top) ΔS_{2n} as a function of even neutron number for $Z = 18$ isotopes; (bottom) ΔS_{2p} as a function of even proton number for $N = 12$ isotones.

The top panel of Fig. 3 shows the ΔS_{2n} variation for $Z = 18$ isotopes and the bottom panel shows the ΔS_{2p} variation for $N = 12$ isotones. Two vertical lines at $N = 14$ and 30 in the top panel and a vertical line at $Z = 16$ in the bottom panel are marked in Figure 3. Figure 3 compares the values estimated using three different mass tables namely AME2003, AME2012 and AME2016 represented using black solid, red dashed and blue dotted line respectively. At the indicated nucleon numbers, estimated from the three mass tables shows a significant variation with respect to each other.

For $Z = 18$ isotopes, AME2012 and AME2016 predicts a strong shell closure at $N = 14$ where $\Delta^2 S_{2n} < -0.5$ MeV indicating a significant drop in ΔS_{2n} value while AME2003 predicts a weak shell closure with $-0.5 < \Delta^2 S_{2n}\,(\text{MeV}) < 0$. On the other hand, among $Z = 18$ isotopes at $N = 30$, AME2003 exhibits a new magicity while AME2016 shows no sign of magicity. The different behaviour of ΔS_{2p} systematics is observed at $Z = 16$ among $N = 12$ isotone. Though AME2012 records new proton magicity at $Z = 16$, but AME2003 predicts no new proton shell closure. Thus the present calculation on two nucleon separation energy difference systematics depends on the type of mass table and the result is sensitive to the update in the mass table.

Summary

The separation energy difference systematics was used as a probe to study the shell evolution in light nuclei. The family of available isotopes of nuclei with charge number in the range of $8 \le Z \le 22$ and isotones with neutron number $8 \le N \le 22$ are studied in the present work. The two nucleon separation energy was calculated using the modified Bethe−Weizsäcker binding energy formula proposed by Samanta and Adhikari with an alteration in asymmetry and pairing energy terms. The experimental two nucleon separation energy was retrieved from AME2016 table. A sharp decrease in separation energy difference was identified as shell closure. Appearance of new magic numbers, disappearance and existence of classic magic numbers were identified from the separation energy difference systematics. Update in the mass table has a significant influence on the separation energy difference systematics as the study relies on the experimental separation energy values.

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