Impact of Nuclear Deformation on Neutron Dripline Prediction: A Study of Mg Isotopes

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ABSTRACT

We have employed the relativistic Hartree-Bogoliubov (RHB) model with density-dependent meson-exchange interaction and separable pairing to investigate neutron dripline mechanisms for heavy Mg isotopes. In the present study, ^{40}Mg is predicted to be dripline nuclei. The calculations are carried out by taking axial deformation into account. An investigation of shape transition is also done for even-even $^{32-42}$Mg isotopes. Our prediction for neutron dripline for ^{40}Mg is consistent with some recent studies.

1. Introduction

The study of nuclei far from β-stability line is a devoted part of modern nuclear-structure studies. The limit of nuclear existence is reached while going away from the stability line. Nuclear drip lines form the edges of the nuclear chart and are defined as limits beyond which a single nucleon become unbound in the nuclear ground-state. The advancement in Radioactive Ion Beam (RIB) facilities and sensitive detection technologies have made it accessible to reach neutron drip lines experimentally. Although the construction of RIBs brings the goal closer, the nuclear drip lines have not been fully mapped experimentally. Moreover, the theoretical location of the nuclear drip line is uncertain [1, 2]. According to a recent study done by Tsunoda et al. [3], the mechanism responsible for the drip line is related to deformation. However, the neutrons can't be added continuously as the atomic nucleus is characterized by a specific number of protons and neutrons and is bounded by drip lines [4]. Earlier, it was assumed that the strong nuclear force for pn symmetry governed the neutron drip line. But protons and neutrons jump from shell to shell, as they do not stay in a single shell, and move collectively by forming different configurations. These configurations produce deformed shapes. Motivating from this idea, we have done a drip line study of Mg isotopes using relativistic Hartree-Bogoliubov (RHB) model with density-dependent meson

exchange (DD-ME2) interaction. Our results for ground-state properties and drip line prediction are consistent with recent theoretical studies. For Mg isotopes, different predictions for neutron drip lines have been proposed. According to the study done in Refs. [5, 6], the heaviest magnesium isotope observed is ^{40}Mg while, the dripline is predicted at ^{42}Mg in some recent studies [3, 7].

Nuclear density functional theories have been using to understand nuclear many-body dynamics for an appreciable description of nuclei near the drip lines [8-11]. It is also found to provide appreciable results for the shape of the nuclei. For the calculations of the shape evolution and ground-state properties, we have used the DIRHBZ numerical code developed by Niksic and others [12]. The present calculations infer the neutron drip line for ^{40}Mg.

2. Theoretical Framework

Self-consistent mean-field (SCMF) models provide a very successful tool to study nuclear ground-state and excited-state properties from the valley of β-stability to the nuclear drip-lines throughout the Segre chart (nuclear chart). These models are based on the nuclear energy density functionals (EDF) in which the nucleons are treated as independent particles moving inside the nucleus under the influence of potentials that are derived from such functionals [13].

2.1. Meson Exchange Models

The detailed description of density dependent meson exchange (DD-ME2) interaction can be found in Ref. [14]. Here, we provide a brief outline of DD-ME2 interaction. The total Lagrangian density of mesons exchange model can be written as:

$$
\begin{aligned}
\mathcal{L} = &\sum_i \bar{\psi}_i \left(i\gamma_\mu \partial^\mu - m \right)\psi_i + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}m_\sigma^2 \sigma^2 - \frac{1}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} \\
&+ \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&- g_\sigma \bar{\psi}\psi\sigma - g_\omega \bar{\psi}\gamma^\mu \psi\omega_\mu - g_\rho \bar{\psi}\vec{\tau}\gamma^\mu \psi\cdot\vec{\rho}_\mu - e\bar{\psi}\gamma^\mu \psi A_\mu,
\end{aligned}
$$

$$(1)$$

where m denotes the bare mass of nucleon and ψ denotes the Dirac spinors. m_σ, m_ω, m_ρ are masses of σ, ω, ρ mesons with corresponding coupling constants g_σ, g_ω, g_ρ for the mesons to the nucleons, respectively. $\Omega_{\mu\nu}$, $\vec{R}_{\mu\nu}$, $F_{\mu\nu}$ are field tensor of the vector fields ω, ρ, and the photon. Arrows denote isovectors and boldface symbols are used for vectors in ordinary space.

The functionals are described by density-dependent coupling constants $g_i(\rho)$ (for $i = \sigma, \omega, \rho, \delta$). The coupling of σ field and ω field to the nucleon field is given by [15-17]

$$
g_i(\rho) = g_i(\rho_{sat})f_i(x) \qquad \text{for } i = \sigma, \omega \qquad (2)
$$

with

$$
f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \qquad (3)
$$

which is a function of $x = \rho/\rho_{sat}$. Here, ρ_{sat} is the baryon density at saturation in symmetric nuclear matter.

For density dependence of ρ-meson coupling, Dirac-Brueckner suggested the functional form [18], given by

$$
g_\rho(\rho) = g_\rho(\rho_{sat})e^{-a_\rho(x-1)} \qquad (4)
$$

The isovector channel is parameterized by $g_\rho(\rho)$ and a_ρ.

2.2. Relativistic Hartree-Bogoliubov Approximation with a Separable Pairing Interaction

The inclusion of pairing correlations are important for a quantitative description of open shell nuclei [19, 20]. The RHB model gives a unified description of particle-hole (ph) and particle-particle (pp) correlations on a mean-field level by using the average self-consistent mean-field potential that encloses the long-range ph correlations and a pairing field potential which sums up the pp correlations. The density

matrix in the presence of pairing can be generalized to two densities, the normal density $\hat{\rho}$, and pairing tensor $\hat{k}$. The relativistic Hartree-Bogoliubov energy density functional can be written as:

$$
E_{RHB}\left[\hat{\rho}, \hat{k}\right] = E_{RMF}\left[\hat{\rho}\right] + E_{pair}\left[\hat{k}\right], \qquad (5)
$$

where $E_{RMF}\left[\hat{\rho}\right]$ is the nuclear energy density functional and is given by

$$
E_{RMF}\left[\psi, \bar{\psi}, \sigma, \omega^\mu, \vec{\rho}^\mu, A^\mu\right] = \int d^3r \mathcal{H}(r) \qquad (6)
$$

The pairing part of RHB functional is given by

$$
E_{pair}\left[\hat{k}\right] = \frac{1}{4}\sum_{n_1 n_1'}\sum_{n_2 n_2'} k^*_{n_1 n_1'}\left\langle n_1 n_1' \left| V^{PP} \right| n_2 n_2' \right\rangle k_{n_2 n_2'}, \qquad (7)
$$

where $\left\langle n_1 n_1' \left| V^{PP} \right| n_2 n_2' \right\rangle$ are the matrix elements of the two-body pairing interaction, and indices n_1, n_1', n_2 and n_2' denote quantum numbers that specify the Dirac indices of the spinor.

The pairing force is separable in momentum space and in r-space has the form of

$$
V^{PP}\left(r_1, r_2, r_1', r_2'\right) = -G\delta\left(R - R'\right)P(r)P(r'), \qquad (8)
$$

where $R = \frac{1}{\sqrt{2}}(r_1 + r_2)$ and $r = \frac{1}{\sqrt{2}}(r_1 - r_2)$ represent the center of mass and the relative coordinates, respectively and the form factor $P(r)$ is of Gaussian form written as,

$$
P(r) = \frac{1}{\left(4\pi a^2\right)^{3/2}} e^{-r^2/2a^2}. \qquad (9)
$$

The two parameters 'G' and 'a' have been adjusted to reproduce the density dependence of gap at the Fermi surface. In the present work, the values of these parameters are taken as G = 728 MeV fm³ and a = 0.644 fm for protons and neutrons. They are calculated with the DIS Gogny force in Refs. [19, 21-23]. The pairing force has a finite range and also it conserves translational invariance due to the presence of the factor $\delta(R - R')$. Finally, the pairing energy in the nuclear ground-state is given by

$$
E_{pair} = -G\sum_N P_N^* P_N \qquad (10)
$$

3. Results and Discussion

Shape is among the most fundamental properties of nuclei and is reflected by the value of the quadrupole deformation parameter. In the axially symmetric case, the deformed

nuclei can be classified depending upon the value of quadrupole deformation parameter (β_2). A positive value of β_2 corresponds to the prolate shape and the negative value of β_2 reflects oblate and $\beta_2 = 0$ corresponds to spherical shape. Fig. 1 presents the potential energy curves (PECs) for $^{32-42}$Mg isotopes calculated by constraining the axial deformation parameter. The nuclei ^{32}Mg have a spherical shape which is due to the presence of $N = 20$ shell-closure. It can be observed from the figure that almost all neutron-rich Mg isotopes have a prolate shape. The deformation is observed to increase as one move towards heavier isotopes and become saturate and then decreases for ^{42}Mg.
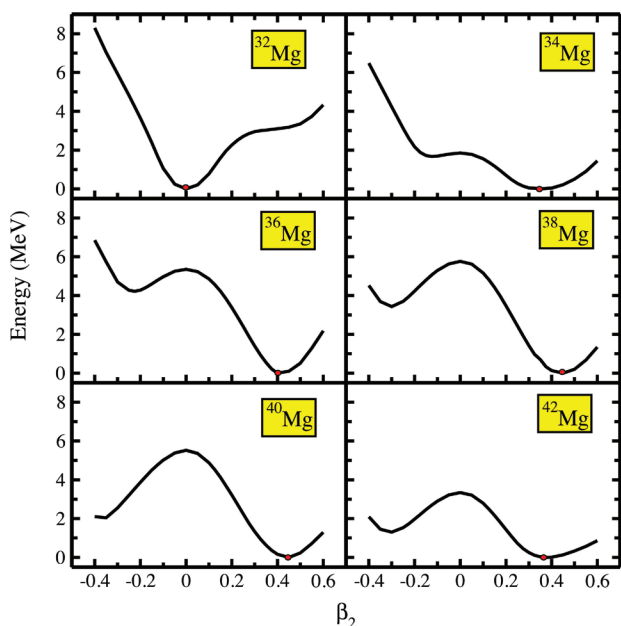
Figure 1: The potential energy curves (PEC), as a function of β_2, for $^{32-42}$Mg isotopes. The energies are normalized with respect to the binding energy of the global minima.

In Fig. 2, we plot the binding energy per nucleon E/A, the neutron chemical potential λ_n, the ground-state quadrupole deformation parameter (β_2), and two-neutron separation energies S_{2n} for neutron-rich Mg isotopes as function of the mass number A. In the top panel of Fig. 2, we observed a decrease in the E/A value indicating the loosely bound behavior of heavier Mg isotopes. In panel (b) of Fig. 2, the neutron chemical potential λ_n is plotted as a function of A. The systematics of λ_n can be related to the large deformation. The negative value of chemical potential assures that an even-N system is bound. The positive value of λ_n for ^{42}Mg indicates an unbound system. The ground-state quadrupole deformation parameter, obtained from PECs, for $^{32-42}$Mg can be depicted in panel (c) of Fig. 2. Moving away from $N = 20$ shell closure, all the neutron-rich Mg isotopes have a prolate shape. The primary factor that determines the stability of the

particle and the nucleoid drip line is two-neutron separation energy S_{2n} [4]. It is defined as the amount of energy needed to remove two neutrons. The stability and instability of the nucleus to nucleon emission can be related to the positive value of S_{2n} and negative value of S_{2n} respectively. The drip line is reached when $S_{2n} \approx 0$. In the bottom panel of Fig. 2, we compared the results of two-neutron separation energies S_{2n}, calculated with difference in binding energies ($S_{2n}(Z, N) = E_b(Z, N) - E_b(Z, N - 2)$) and with the related quantity ($S_{2n} = -2\lambda_n$) [4]. A negative value of S_{2n} for ^{42}Mg shows it to be an unbound nucleus.

The variation of nuclear shape and ground-state energy are presented, as a function of neutron number (N), in Fig. 3. The nucleus ^{32}Mg has a spherical shape and have lower binding energy. The evolution of ellipsoidal (prolate in the present case) deformation is observed with the addition of the number of neutrons and an increase in binding energy is seen from ^{32}Mg to ^{40}Mg. With the increase in neutron number, the superposition of single-particle states of pf shell leads to enhance the deformation, giving the nucleus additional binding energy. At $N = 28$, this additional binding energy saturates and decreases after adding a pair of neutron to the nucleus. Thus, after this point, the deformation gets weaker by the addition of two neutrons and they drip off the nuclei.
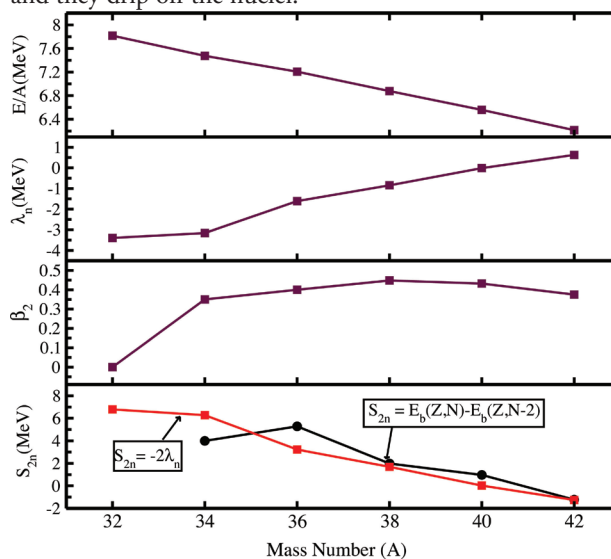

Figure 2: Binding energies per nucleon E/A, neutron chemical potentials λ_n, quadrupole deformation parameter (β_2), and two-neutron separation energies S_{2n} for neutron-rich Mg isotopes plotted as function of the mass number A.

Conclusion

In this paper, we have performed axially constrained self-consistent calculations using relativistic Hartree-Bogoliubov (RHB) model with density-dependent meson-exchange

interaction (DD-ME2) and with separable pairing. We have studied the shape evolution of some neutron-rich Mg isotopes and calculated their ground-state properties. The negative value of two-neutron separation energies for ^{42}Mg correspond to an unbound system. The heaviest isotope that is bound against two neutron decay, based on positive S_{2n} is ^{40}Mg. Our calculations predict ^{40}Mg to be a drip line nucleus, which is consistent with recent theoretical predictions. The influence of deformation for the prediction of neutron drip line for Mg isotopes is also studied.
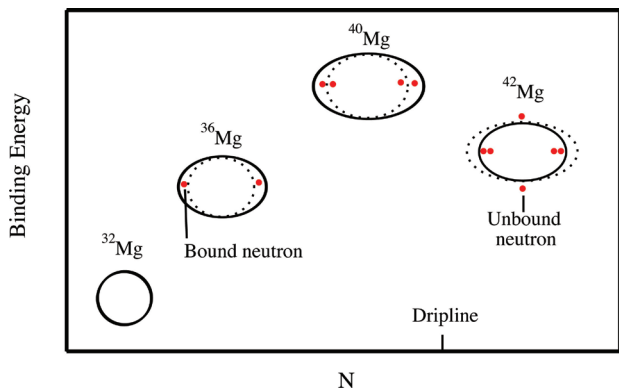
Figure 3: Variation of nuclear shape and ground-state energy, as a function of neutron number (N).

Acknowledgements

References

[1] A. V. Afanasjev, S. E. Agbemava, D. Ray and P. Ring, Phys. Rev. C **91**, 014324 (1992). https://doi.org/10.1103/PhysRevC.91.014324

[2] L. Neufcourt et al., Phys. Rev. Lett. **122**, 062502 (2019). https://doi.org/10.1103/PhysRevLett.122.062502

[3] N. Tsunoda et al., Nature **587**, 66 (2020). https://doi.org/10.1038/s41586-020-2848-x

[4] J. Erler et al., Nature **486**, 509 (2012). https://doi.org/10.1038/nature11188

[5] T. Baumann, et al., Nature **449**, 1022 (2007). https://doi.org/10.1038/nature06213

[6] H. L. Crawford et al., Phys. Rev. Lett **122**, 052501 (2014). https://doi.org/10.1103/PhysRevLett.122.052501

[7] Q. Chai, J. Pei, N. Fei and D. Guan, Phys. Rev. C **102**, 014312 (2020). https://link.aps.org/doi/10.1103/PhysRevC.102.014312

[8] V. Thakur and S. K. Dhiman, Nucl. Phys. A **992**, 121623 (2019). https://doi.Org/10.1016/j.nuclphysa.2019.121623

[9] V. Thakur Nuet et al., Nucl. Phys. A **1002**, 121981 (2020). https://doi.Org/10.1016/j.nuclphysa.2020.121981

[10] P. Kumar and S. K. Dhiman, Nucl. Phys. A **1001**, 121935 (2020). https://doi.Org/10.1016/j.nuclphysa.2020.121935

[11] P. Kumar et al., Eur. Phys. J. A **57**, 36 (2021). https://doi.org/10.1140/epja/s10050-021-00346-6

[12] T. Niksic, N. Paar, D. Vretenar and P. Ring, Computer Physics Communications **185**, 1808 (2014). https://doi.Org/10.1016/j.cpc.2014.02.027

[13] M. Bender, P.-H. Heenen and P.-G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003). https://doi.org/10.1103/RevModPhys.75.121

[14] G. Lalazissis, T. Niksic, D. Vretenar and P. Ring, Phys. Rev. C **71**, 024312 (2005). https://doi.org/10.1103/PhysRevC.71.024312

[15] S. Typel and H. H. Wolter, Nucl. Phys. A **656**, 331 (1999). https://doi.org/10.1016/S0375-9474(99)00310-3

[16] F. Hofmann, C. Keil and H. Lenske, Phys. Rev. C **64**, 034314 (2001). https://doi.org/10.1103/PhysRevC.64.034314

[17] T. Niksic, D. Vretenar, P. Finelli and P. Ring, Phys. Rev. C **66**, 024306 (2002). https://doi.org/10.1103/PhysRevC.66.024306

[18] F. De Jong and H. Lenske, Phys. Rev. C **57**, 3099 (1998). https://doi.org/10.1103/PhysRevC.57.3099

[19] Y. Tian, Z.-Y. Ma and P. Ring, Phys. Lett. B **676**, 44 (2009). https://doi.org/10.1016/j.physletb.2009.04.067

[20] T. Niksic, P. Ring, D. Vretenar, Y. Tian and Z.-Y. Ma, Phys. Rev. C **81**, 054318 (2010). https://doi.org/10.1103/PhysRevC.81.054318

[21] Y. Tian, Z.-Y. Ma and P. Ring, Phys. Rev. C **79**, 064301 (2009). https://doi.org/10.1103/PhysRevC.79.064301