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# Spontaneous *CP* Violation Jarlskog Invariant in $SM \otimes S_3$

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ABSTRACT

In our days, *CP* (Charge Parity) violation in the Standar Model of fundamental interactions still remains as an open problem. It is well known that explicit *CP* violation may be included by impossing complex Yukawa couplings in the Yukawa sector or complex Higgs couplings in exttended Higgs sectors with more than one Higgs field. It is desirable to have a fundamental *CP* violation theory, in that sence, we analyse the diferent secenarios for Spontaneous *CP* violation in an exteded Higgs model with three Higgs fields and a discrete flavour permutational symmetry $S_3$. Spontaneous *CP* violation effects contribute to the Higgs mass matrix, as well as, up and down quark mass matrices. This complex quark mass matrices allow us to study the conditions for a non-vanishing Jarlskog invariant *J* which provides a necessary and sufficient contribution for a spontaneous *CPV* coming from $SM \otimes S_3$

## 1. Introduction

Although highly successful in terms of its phenomenological predictions, the Standard Model (*SM*) of electroweak interactions seems incomplete from a theoretical view. In its present form, it is unable to predict the masses of fermions (leptons and quarks), or explain why there are several families of such particles. In the *SM*, only one $SU(2)_L$ doublet Higgs field is included, which, upon acquiring a vacuum expectation value, breaks the $SU(2)_L \times U(1)_Y$ symmetry. In the *SM* each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory. An extended Higgs sector opened up the window for *CP* violation scenarios coming from the Higgs sector, we look for the conditions under which *CP* violation arises from spontaneous gauge symmetry breaking $SU(2)_L \times U(1)_Y \xrightarrow{SB} U(1)_{EM}$. In this direction interesting work has been done with the addition of discrete symmetries to the *SM*. It is noticeable that many interesting features of masses and mixing of the *SM* can be understood using a minimal discrete group, namely the permutational group $S_3$.

## 2. Higgs Sector in $SM \otimes S_3$

The Lagrangian $L_\phi$ of the Higgs sector is given by

$$L_\phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_s) \quad (1)$$

where $D_\mu$ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_s)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3$. The analysis of the stability properties of the Higgs potential $V$ is of great relevance to study the phenomenological implications of this model. There are many dif ferent ways of writing the Higgs potential for this model, but for the purpose of this work the best basis is

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}. \quad (2)$$

### *2.1 The Higgs Potential*

Is usual to write the potential in therms of the invariants $x_i$,

$$\begin{aligned} x_1 &= H_1^{T*} H_1, & x_4 &= R\left(H_1^{T*} H_2\right), & x_7 &= I\left(H_1^{T*} H_2\right), \\ x_2 &= H_2^{T*} H_2, & x_5 &= R\left(H_1^{T*} H_S\right), & x_8 &= I\left(H_1^{T*} H_S\right), \\ x_3 &= H_S^{T*} H_S, & x_6 &= R\left(H_2^{T*} H_S\right), & x_9 &= I\left(H_2^{T*} H_S\right). \end{aligned} \quad (3)$$


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considering our assignment, the Higgs potential [1-4] becomes

$$\begin{aligned} V = \mu_1^2\left(x_1+x_2\right)+\mu_0^2 x_3+a x_3^2+b\left(x_1+x_2\right)x_3 \\ +c\left(x_1+x_2\right)^2-4dx_7^2+2e\left[\left(x_1-x_2\right)x_6-2x_4x_5\right] \\ +f\left(x_5^2+x_6^2+x_8^2+x_9^2\right)+g\left[\left(x_1+x_2\right)^2+4x_4^2\right] \\ +2h\left(x^2+x_6^2-x_8^2-x_9^2\right) \end{aligned} \quad (4)$$

where the $\mu_{0,1}^2$ parameters have dimensions of mass squared, the *a, b, c, d, e, f, g, h,* parameters are dimensionless.

## 3. Stationary Points

The stationary points of the potential (4) where computed in order to determine their phenomenological feasibility [1]. We assume here that $H_S$ is the *SM* Higgs, in the minimum that we are interested, *CP* is violated but we do not break the electric charge symmetry when the Gauge symmery is broken down to $U(1)_Y$. If there is *CP* breaking in the model we are assuming that it is due to any of the three $S_3$ Higgs doublet fields. Taking this into account, and using the minimization conditions, we find that the potential (4) has a *CP* breaking stationary point. For the *CP* breaking minimum we have non vanishing *VEVs*

$$\begin{aligned} \phi_7=v_1,\quad \phi_8=v_2,\ \phi_9=v_3,\ \phi_{10}=\gamma_1, \\ \phi_{11}=\gamma_2,\ \phi_{12}=\gamma_3, \end{aligned} \quad (5)$$

for other fields we have $\phi_i = 0$.

### 3.1 Spontaneous CP Breaking

In the *CP* breaking minimum (*CPB*) [2] we have a contribution from the three Higgs doublet fields

$$\left\langle\Phi_i\right\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_i+i\gamma_i\end{pmatrix} \qquad i=1,2,3, \quad (6)$$

where $\gamma_i \in \Re$. Then, *CPB* is at $\phi_7=v_1$, $\phi_8=v_2$, $\phi_9=v_3$, $\phi_{10}=\gamma_1$, $\phi_{11}=\gamma_2$, $\phi_{12}=\gamma_3$ and other cases $\phi_i=0$, which should satisfy the constraint

$$v_1^2+v_2^2+v_3^2+\gamma_1^2+\gamma_2^2+\gamma_3^2=v^2 \quad (7)$$

To complete the story, the *VEVs* $\gamma_\iota$ allowes *CP* breaking scenarios with a contribution coming from one, two or three Higgs fields:

| *Scenario* | $\phi_7$ | $\phi_8$ | $\phi_9$ | $\phi_{10}$ | $\phi_{11}$ | $\phi_{12}$ |
|---|---|---|---|---|---|---|
| *I* | $v_1$ | $v_2$ | $v_3$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
| *II* | $v_1$ | $v_2$ | $v_3$ | $\gamma_1$ | 0 | 0 |
| *III* | $v_1$ | $v_2$ | $v_3$ | 0 | $\gamma_2$ | 0 |
| *IV* | $v_1$ | $v_2$ | $v_3$ | 0 | 0 | $\gamma_3$ |
| *V* | $v_1$ | $v_2$ | $v_3$ | $\gamma_1$ | $\gamma_2$ | 0 |
| *VI* | $v_1$ | $v_2$ | $v_3$ | $\gamma_1$ | 0 | $\gamma_3$ |
| *VII* | $v_1$ | $v_2$ | $v_3$ | 0 | $\gamma_2$ | $\gamma_3$ |

The minimization conditions $\left(\partial V/\partial\phi_i\right)\big|_{\min}=0$ give us six equations with more free parameters than independent equations in any scenario. We focus in Higgs *VEVs* allowed range of values to compute the Up and Down quark mass matrices and the Jarlskog invariant. In Scenario I, we have the following minimum equations:

$$\begin{aligned} 0 = 2\left(c+g\right)v_1^3+2\gamma_1\left(v_2\left(2\left(d+g\right)\gamma_2+e\gamma_3\right)+v_3\left(e\gamma_2+2h\gamma_3\right)\right) \\ +v_1(2\left(c+g\right)v_2^2+6ev_2v_3+\left(b+f+2h\right)v_3^2+2\left(c+g\right)\gamma_1^2 \\ +\left(b+f-2h\right)\gamma_3^2+2\gamma_2\left(\left(c-2d-g\right)\gamma_2+e\gamma_3\right)+\mu_1^2), \end{aligned} \quad (8)$$

$$\begin{aligned} 0 = 2\left(c+g\right)v_2^3-3ev_2^2v_3\ +v_1^2(2\left(c+g\right)v_2+3ev_3) \\ +2v_1\gamma_1\left(2\left(d+g\right)\gamma_2+e\gamma_3\right)+2v_3\left(e\gamma_1^2+\left(4h\gamma_3-e\gamma_2\right)\gamma_2\right) \\ +v_2(\ \left(b+f+2h\right)v_3^2+2\left(c-2d-g\right)\gamma_1^2+2\left(c+g\right)\gamma_2^2 \\ -2e\gamma_2\gamma_3+\left(b+f-2h\right)\gamma_3^2+\mu_1^2), \end{aligned} \quad (9)$$

$$\begin{aligned} 0 = -ev_2^3+\left(b+f-2h\right)v_2^2v_3+(3ev_2+\left(b+f+2h\right)v_3)v_1^2 \\ +2v_1\gamma_1\left(e\gamma_2+2h\gamma_3\right)+v_2\left(e\gamma_1^2-e\gamma_2^2+4h\gamma_2\gamma_3\right) \\ +v_2\left(\left(b+f-2h\right)(\gamma_1^2+\gamma_2^2)+2a(\gamma_3^2+v_3^2)+\mu_0^2\right), \end{aligned} \quad (10)$$

$$\begin{aligned} 0 = 2\left(c+g\right)v_1^2\gamma_1+2v_1\left(v_2\left(2\left(d+g\right)\gamma_2+e\gamma_3\right)+v_3\left(e\gamma_2+2h\gamma_3\right)\right) \\ +\gamma_1(2\left(c-2d-g\right)v_2^2+2ev_2v_3+\left(b+f-2h\right)v_3^2 \\ +2\left(c+g\right)(\gamma_1^2+\gamma_2^2)+6e\gamma_2\gamma_3+\left(b+f+2h\right)\gamma_3^2+\mu_1^2), \end{aligned} \quad (11)$$

$$\begin{aligned} 0 = 2v_1\left(2\left(d+g\right)v_2+ev_3\right)\gamma_1+\left(b+f-2h\right)v_3^2\gamma_2 \\ +2\left(c+g\right)(\gamma_1^2+\gamma_2^2)\gamma_2+3e(\gamma_1^2-\gamma_2^2)\gamma_3+\left(b+f+2h\right)\gamma_2\gamma_3^2 \\ +v_2^2\left(2\left(c+g\right)\gamma_2-e\gamma_3\right)+2\left(c-2d-g\right)v_1^2\gamma_2+ev_1^2\gamma_3 \\ -2v_2v_3\left(e\gamma_2-2h\gamma_3\right)+\gamma_2\mu_1^2, \end{aligned} \quad (12)$$

$$\begin{aligned} 0 = 2v_1\left(ev_2+2hv_3\right)\gamma_1+4hv_2v_3\gamma_2 \\ +e(3\gamma_1^2-\gamma_2^2)\gamma_2+2a(v_3^2+\gamma_3^2)\gamma_3 \\ +\left(b+f+2h\right)(v_1^2+v_2^2)\gamma_3+e(v_1^2-v_2^2)\gamma_2+\gamma_3\mu_0^2. \end{aligned} \quad (13)$$

In each scenario we obtain conditions to constrain the allowed *VEVs*, that fulfills the minimum equations.

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### *3.2 The CP Breaking Minimum with Polar Phase.*

In order to compare with other authors [2], the *VEVs* in the *CPB* can be written as follows

$$\langle H_1\rangle=\begin{pmatrix}0\\ N_1e^{i\theta_1}\end{pmatrix},\langle H_2\rangle=\begin{pmatrix}0\\ N_2e^{i\theta_2}\end{pmatrix},\langle H_S\rangle=\begin{pmatrix}0\\ N_3e^{i\theta_3}\end{pmatrix}. \quad (14)$$

where

$$N_1=\sqrt{v_1^2+\gamma_1^2},\ N_2=\sqrt{v_2^2+\gamma_2^2},\ N_3=\sqrt{v_3^2+\gamma_3^2}, \\ \frac{1}{2}\tan\theta_1=\frac{\gamma_1}{v_1},\ \tan\theta_2=\frac{\gamma_2}{v_2},\ \tan\theta_3=\frac{\gamma_3}{v_3}. \quad (15)$$

The potential minimization conditions give us six equations

$$\begin{aligned}0=&2N_1(2(c+g)N_1^2)+2(c-d+(d+g)Cos(2(\theta_1-\theta_2)))N_2^2\\&+2e(Cos(2\theta_1-\theta_2-\theta_3)+2Cos(\theta_2-\theta_3))N_2N_3\\&+(b+f+2hCos(2(\theta_1-\theta_3)))N_3^2+\mu_1^2),\end{aligned} \quad (16)$$

$$\begin{aligned}0=&2(N_1^2\begin{pmatrix}2(c-d+(d+g)Cos(2(\theta_1-\theta_2)))N_2\\ +e(Cos(2\theta_1-\theta_2-\theta_3)+2Cos(\theta_2-\theta_3))N_3\end{pmatrix}\\&+N_2\begin{pmatrix}2(c+g)N_2^2-3eCos(\theta_2-\theta_3)N_2N_3\\ +(b+f+2hCos(2(\theta_2-\theta_3)))N_3^2+\mu_1^2\end{pmatrix},\end{aligned} \quad (17)$$

$$\begin{aligned}0=&2(-eCos(\theta_2-\theta_3)N_2^3+(b+f+2hCos(2(\theta_2-\theta_3)))N_2^2N_3\\&+(2aN_3^2+\mu_0^2)N_3\\&+N_1^2\begin{pmatrix}e(Cos(2\theta_1-\theta_2-\theta_3)+2Cos(\theta_2-\theta_3))N_2\\ +(b+f+2hCos(2(\theta_1-\theta_3)))N_3\end{pmatrix},\end{aligned} \quad (18)$$

$$0=-4N_1^2\begin{pmatrix}(d+g)Sin(2(\theta_1-\theta_2))N_2^2+eSin(2\theta_1-\theta_2-\theta_3)N_2N_3\\ +hSin(2(\theta_1-\theta_3))N_3^2\end{pmatrix}, \quad (19)$$

$$\begin{aligned}0=&2N_2(Sin(\theta_2-\theta_3)N_2N_3(eN_2-4hN_3Cos(\theta_2-\theta_3))\\&+N_1^2\left(2(d+g)Sin(2(\theta_1-\theta_2))N_2+e\begin{pmatrix}Sin(2\theta_1-\theta_2-\theta_3)\\ -2Sin(\theta_2-\theta_3)\end{pmatrix}N_3\right),\end{aligned} \quad (20)$$

$$\begin{aligned}0=&2N_3(Sin(\theta_2-\theta_3)N_2^2(-eN_2+4hN_3Cos(\theta_2-\theta_3))\\&+N_1^2\begin{pmatrix}e(Sin(2\theta_1-\theta_2-\theta_3)+2Sin(\theta_2-\theta_3))N_2\\ +2hSin(2(\theta_1-\theta_3))N_3\end{pmatrix}.\end{aligned} \quad (21)$$

If we rotate the phases in such a way that $\sigma_1=\theta_1-\theta_3$ and $\sigma_2=\theta_2-\theta_3$, the minimum conditions are written as follows:

$$\begin{aligned}0=&2N_1(2(c+g)N_1^2)\\&+2\left(c-d+(d+g)Cos\left(2\left(\sigma_1-\sigma_2\right)\right)\right)N_2^2\\&+2e\left(Cos\left(2\sigma_1-\sigma_2\right)+2Cos\left(\sigma_2\right)\right)N_2N_3\\&+\left(b+f+2hCos\left(2\sigma_1\right)\right)N_3^2+\mu_1^2),\end{aligned} \quad (22)$$

$$\begin{aligned}0=&2(N_1^2\begin{pmatrix}2(c-d+(d+g)Cos(2(\sigma_1-\sigma_2)))N_2\\ +e(Cos(2\sigma_1-\sigma_2)+2Cos(\sigma_2))N_3\end{pmatrix}\\&+N_2\begin{pmatrix}2(c+g)N_2^2-3eCos(\sigma_2)N_2N_3\\ +(b+f+2hCos(2\sigma_2))N_3^2+\mu_1^2\end{pmatrix},\end{aligned} \quad (23)$$

$$\begin{aligned}0=&2(-eCos(\sigma_2)N_2^3+(b+f+2hCos(2\sigma_2))N_2^2N_3\\&+(2aN_3^2+\mu_0^2)N\\&+N_1^2\begin{pmatrix}e(Cos(2\sigma_1-\sigma_2)+2Cos(\sigma_2))N_2\\ +(b+f+2hCos(2\sigma_1))N_3\end{pmatrix},\end{aligned} \quad (24)$$

$$0=-4N_1^2\begin{pmatrix}(d+g)Sin(2(\sigma_1-\sigma_2))N_2^2+eSin(2\sigma_1-\sigma_2)N_2N_3\\ +hSin(2\sigma_1)N_3^2\end{pmatrix}, \quad (25)$$

$$\begin{aligned}0=&2N_2(Sin(\sigma_2)N_2N_3(eN_2-4hN_3Cos(\sigma_2))\\&+N_1^2\left(2(d+g)Sin(2(\sigma_1-\sigma_2))N_2+e\begin{pmatrix}Sin(2\sigma_1-\sigma_2)\\ -2Sin(\sigma_2)\end{pmatrix}N_3\right),\end{aligned} \quad (26)$$

$$\begin{aligned}0=&2N_3(Sin(\sigma_2)N_2^2(-eN_2+4hN_3Cos(\sigma_2))\\&+N_1^2(e(Sin(2\sigma_1-\sigma_2)+2Sin(\sigma_2))N_2+2hSin(2\sigma_1)N_3).\end{aligned} \quad (27)$$

Unlike reference [3], we have an additional minimum equation, so we will continue working with the first representation in section 3.1.

## 4. Jarlskog Invariant

The formalism of the Jarlskog invariant imposes conditions which must be satisfied in order to have *CP* violation [5]

$$m_u\neq m_c,\ m_c\neq m_t,\ m_t\neq m_u,\ m_d\neq m_s,\ m_s\neq m_b, \quad (28)$$

$$m_b\neq m_d,\ \delta\neq 0,\pi,\ \theta_j\neq 0,\frac{\pi}{2},\quad j=1,2,3$$

where $m_i$ are respective quark mass value. These conditions are unified within the single relation $\det C\neq 0$ where

$$iC=\left[M^uM^{uT*},M^dM^{dT*}\right] \quad (29)$$

Where $M^u$ and $M^d$ denotes the Up and Down quark mass matrices. What is highly remarkable about the above commutator is that its determinant is given by

$$\begin{aligned}\det C=&-2J\left(m_t^2-m_c^2\right)\left(m_c^2-m_u^2\right)\left(m_u^2-m_t^2\right)\\&\left(m_b^2-m_s^2\right)\left(m_s^2-m_d^2\right)\left(m_d^2-m_b^2\right)\end{aligned} \quad (30)$$

where $J$ is the Jarlskog invariant. $J$ vanishes in the absence of $CPB$ and viceversa for non vanishing $J$.

### 4.1 The Fermionic Mass Matrix

The Yukawa Lagangian of the extended model with three Higgs doublets [6-9] can be written as

$$L_Y = L_{Y_D} + L_{Y_U} + L_{Y_E} + L_{Y_\nu}$$

Each term is given as

$$\begin{aligned} L_{Y_D} = & -Y_1^d \overline{Q_I} H_S d_{IR} - Y_3^d \overline{Q_3} H_S d_{3R} \\ & - Y_1^d \left[ \overline{Q_I} \kappa_{IJ} H_1 d_{JR} + \overline{Q_I} \eta_{IJ} H_2 d_{JR} \right] \\ & - Y_4^d \overline{Q_3} H_I d_{IR} - Y_5^d \overline{Q_I} H_I d_{3R} + H.C., \end{aligned} \tag{31}$$

$$\begin{aligned} L_{Y_U} = & -Y_1^u \overline{Q_I} (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q_3} (i\sigma_2) H_S^* u_{3R} \\ & -Y_1^u \left[ \overline{Q_I} \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \overline{Q_I} \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \right] \\ & -Y_4^u \overline{Q_3} (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q_I} (i\sigma_2) H_I^* u_{3R} + H.C., \end{aligned} \tag{32}$$

$$\begin{aligned} L_{Y_E} = & -Y_1^e \overline{L_I} H_S e_{IR} - Y_3^e \overline{L_3} H_S e_{3R} \\ & - Y_1^e \left[ \overline{L_I} \kappa_{IJ} H_1 e_{JR} + \overline{L_I} \eta_{IJ} H_2 e_{JR} \right] \\ & - Y_4^e \overline{L_3} H_I e_{IR} - Y_5^e \overline{L_I} H_I e_{3R} + H.C., \end{aligned} \tag{33}$$

$$\begin{aligned} L_{Y_\nu} = & -Y_1^\nu \overline{L_I} (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L_3} (i\sigma_2) H_S^* \nu_{3R} \\ & -Y_1^\nu \left[ \overline{L_I} \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L_I} \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \right] \\ & -Y_4^\nu \overline{L_3} (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L_I} (i\sigma_2) H_I^* \nu_{3R} + H.C.. \end{aligned} \tag{34}$$

Singlets carry the index s or 3 and doublets carry indices I; J = 1; 2

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{35}$$

Furthermore, we add mass terms for the Majorana neutrinos

$$L_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R}, \tag{36}$$

where $C$ is the charged matrix. From this, we can express the fermionic mass matrix $M_f$ including spontaneous $CP$ violation as

$$M_f = \begin{pmatrix} m_1 + m_6 & m_2 & m_5 \\ m_2 & m_1 - m_6 & m_8 \\ m_4 & m_7 & m_3 \end{pmatrix}, \tag{37}$$

where

$$\begin{aligned} m_1^f &= -Y_1^f (v_3 + i\gamma_3), & m_2^f &= -Y_2^f (v_1 + i\gamma_1), \\ m_3^f &= -Y_3^f (v_3 + i\gamma_3), & m_4^f &= -Y_4^f (v_1 + i\gamma_1), \\ m_5^f &= -Y_5^f (v_1 + i\gamma_1), & m_6^f &= -Y_4^f (v_2 + i\gamma_2), \\ m_7^f &= -Y_4^f (v_2 + i\gamma_2), & m_8^f &= -Y_5^f (v_2 + i\gamma_2). \end{aligned}$$

Then, we obtain complex fermionic mass matrices caused by contribution arising from the Higgs sector. Thus, the $SSB$ mechanism provides a source for $CP$ violation in the fermionic sector and contributes to both, quark and lepton mixing matrices. Assuming real yukawa parameters, $CP$ violation entirely comes from the Higgs sector.

## 5. Scenario Analysis

For each scenario we have for mass parameter $\mu_0^2$

$$\begin{aligned} \mu_0^2 = & \frac{e v_2^3 - (b+f+2h) v_2^2 v_3 - v_1^2 \left(3 e v_2 + (b+f+2h) v_3\right) - 2 v_1 \gamma_1 \left(e\gamma_2 + 2h\gamma_3\right)}{v_3} \\ & + \frac{v_2 \left(-e\gamma_1^2 + (e\gamma_2 - 4h\gamma_3)\gamma_2\right) - v_3 \left((b+f-2h)(\gamma_1^2 + \gamma_2^2) + 2a(v_3^2 + \gamma_3^2)\right)}{v_3} \end{aligned} \tag{38}$$

for ${\mu_1}^2$, we obtain two solutions choose one of the following

$$\begin{aligned} \mu_1^2 = & -\left(2(c+g) v_2^2 + 6 e v_2 v_3 + (b+f+2h) v_3^2 + 2(c+g)\gamma_1^2 + 2(c-2d-g)\gamma_2^2 + 2e\gamma_2\gamma_3 + (b+f-2h)\gamma_3^2\right) \\ & + \frac{-2(c+g) v_1^3 - 2\gamma_1 \left(v_2 (2(d+g)\gamma_2 + e\gamma_3) + v_3 (e\gamma_2 + 2h\gamma_3)\right)}{v_1} \end{aligned} \tag{39}$$

or this

$$\begin{aligned} \mu_1^2 = & 3 e v_2 v_3 - \left((b+f+2h) v_3^2 + 2(c-2d-g)\gamma_1^2 + 2(c+g)\gamma_2^2 - 2e\gamma_2\gamma_3 + (b+f-2h)\gamma_3^2\right) \\ & + \frac{-2(c+g) v_2^3 - \left(2(c+g) v_2 + 3 e v_3\right) v_1^2 - 2 v_1 \gamma_1 (2(d+g)\gamma_2 + e\gamma_3) + v_3 \left(-e\gamma_1^2 + (e\gamma_2 - 4h\gamma_3)\gamma_2\right)}{v_2} \end{aligned} \tag{40}$$

In each scenario we have regions of the allowed *VEVs* parameter space that satisfy the minimum conditions, but these conditions not necessarily satisfied the Jarlskog commutator formalism. The conditions that satisfy the *CPB* minimum as well as a non vanishing Jarlskog invariant, so we have *CP* violation is presented in all scenarios. In scenario I we have Spontaneous *CP* violation when:

$$1.\ e=0,\quad v_2=\frac{\gamma_2}{\gamma_1}v_1,\quad \gamma_3=-\frac{v_1}{\gamma_1}v_3 \tag{41}$$

$$2.\ e=0,\quad v_1=0,\quad v_2=0,\quad \gamma_3=0 \tag{42}$$

$$3.\ e=0,\quad h=0,\quad v_2=\frac{\gamma_2}{\gamma_1}v_1 \tag{43}$$

In the scenario II we have *CP* violation for

$$e=\pm2\sqrt{(d+g)h},\quad v_3=-2\frac{(d+g)}{e}v_2,\quad v_2=\pm\sqrt{\frac{v_1^2+\gamma_1^2}{3}} \tag{44}$$

In scenario III we have *CP* violation for

$$e=\pm2\sqrt{(d+g)h},\quad v_3=-2\frac{(d+g)}{e}v_2,\quad v_2=\pm\sqrt{\frac{v_1^2-\gamma_2^2}{3}} \tag{45}$$

In scenario IV we have *CP* violation in two different parameter regions

$$1.\ e=0,\quad h=0,\quad v_2=v_1 \tag{46}$$

$$2.\ e=0,\quad v_2=v_1,\quad v_3=0 \tag{47}$$

In scenario V we also have *CP* violation in two different parameter regions

$$1.\ e=0,\quad h=0,\quad v_2=\frac{\gamma_2}{\gamma_1}v_1 \tag{48}$$

$$2.\ e=0,\quad h=\frac{2(d+g)(\gamma_1^2-v_1^2)}{v_3^2}\gamma_1,\ v_2=-v_1,\quad \gamma_2=\gamma_1 \tag{49}$$

In scenario VI we have *CP* violation in four parameter regions

$$1.\ e=0,\quad h=0,\quad v_2=0 \tag{50}$$

$$2.\ d=-g,\quad e=0,\quad h=0,\quad v_2=\gamma_1 \tag{51}$$

$$3.\ e=0,\quad h=0,\quad v_1=0,\quad v_2=\gamma_1 \tag{52}$$

$$4.\ e=0,\quad h=0,\quad v_2=0_1,\quad \gamma_1=0 \tag{53}$$

In the scenario VII we have *CP* violation for

$$e=\pm2\sqrt{(d+g)h},\quad v_1=\pm\sqrt{3\left(v_2^2+\gamma_2^2\right)},\ v_3=\frac{\gamma_3}{\gamma_2}v_2,\quad \gamma_3=-\frac{e}{2h}\gamma_2 \tag{54}$$

In all scenarios the jarlskog invariant is different from zero. In a future work a numerical analysis will be presented elsewhere.

## 6. Conclusions

In this work, we analyzed the *SSB* of $SU(2)\times U(1)\longrightarrow U(1)_{EM}$ in $SM\otimes S_3$ with spontaneous *CPV* provided by the Higgs sector. In this model, we introduced three Higgs *SU*(2) doublets with twelve real fields. While defining the gauge symmetry spontaneous breaking, we found a parameter space region where the minimum of the potential defines a *CPB* ground state. We analyzed seven possible scenarios for spontaneous *CP* violation defined in concordance with the *CPV* source Higgs field.

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