J. Nucl. Phys. Mat. Sci. Rad. A. Vol. 9, No. 2 (2022), pp.215–221

CHITKARA

Journal of Nuclear Physics, Material Sciences, Radiation and Applications
Journal homepage: https://jnp.chitkara.edu.in/

Phase Shift Analysis for Neutron-Alpha Elastic Scattering Using Phase Function Method with Local Gaussian Potential

Lalit Kumar, Anil Khachi and O.S.K.S Sastri*

Department of Physical and Astronomical Sciences, Central University of Himachal Pradesh, Dharamshala, Himachal Pradesh-176215, Bharat(India)

*sastri.osks@hpcu.ac.in (Corresponding Author)

ARTICLE INFORMATION

Received: January 14, 2022
Accepted: June 03, 2022
Published Online: June 20, 2022

Keywords:
n- Scattering, Phase Function Method (PFM), Gaussian potential, Cross-section

DOI: 10.15415/jnp.2022.92032

ABSTRACT

Background: The nucleon-nucleus scattering has been studied using Gaussain potential with spin-orbit term of Thomas type to fit the experimental scattering phase shifts (SPS). Recently, Hulthen potential without spin-orbit term has been utilised for studying α–nucleon scattering with phase function method (PFM).

Purpose: The main objectives of this paper are:

1. To obtain the best possible interaction potentials that best describe the neutron-α elastic SPS in various channels.
2. To compute the partial cross-sections for scattering p-states and the total cross-section for the reaction.

Methods: The local interaction potential is modeled using Gaussian function. The non-local spin orbit term is chosen to be proportional to derivative of local potential. The phase function method has been numerically solved using 5th order Runge-Kutta method to compute the SPS. The model parameters are varied in an iterative fashion to minimise the mean absolute percentage error (MAPE) w.r.t. the experimental SPS.

Results:

1. The SPS for S, P and D channels have been obtained with MAPE values less than 3%.
2. The partial cross-sections for p1/2 and p3/2 have been plotted and the respective resonance energies and FWHM have been found to be in reasonable agreement with values in literature.
3. The total cross-section for the reaction has been determined and found to be matching well with experimental findings.

Conclusions: Gaussian potential with associated spin-orbit term has been shown to be a reasonably good choice for explaining the n-α scattering reaction.

1. Introduction

The phenomenon that play an important role in understanding various nuclear properties is scattering. To highlight the underlying physics behind this scattering experiments, various theoretical models have been proposed. Scattering of light nuclei with nucleons and among themselves plays an important role in understanding the underlying interaction and gives information regarding their structure. The study of α-nucleon elastic scattering as a two-body problem at low energy has some importance in the cluster model [1-5] description of nuclei. An extensive study of alpha-nucleon systems has been done by several groups, where results of elastic scattering phase shifts are reported both from experimental and theoretical point of view [6-11]. Alpha-particles have a number of features that make interpreting studies with them easier. Because the alpha particle has such a stable and symmetric structure, its deformation by the oncoming nucleon in a collision can usually be ignored as a first approximation, and many terms in the equations either add or cancel. Alpha particle have zero spin, so that coupled equations don't arise in the investigation of scattering, and polarization peculiarities are bound to the nucleon. It has no excited states beneath somewhere around 20 MeV. So, at lower energies, there can be no inelastic scattering to complicate the phenomena of scattering.

The generator coordinate method (GCM) or the resonating group method (RGM) [12] were oftenly used to model elastic scattering between light nuclei. The use of phenomenological two-body interactions in this model results in good agreement with experimental data for elastic

©Author(s) 2022. This article is published with open access at https://jnp.chitkara.edu.in.
ISSN No.: 2321-8649(Print) ISSN No.: 2321-9289(Online); Registration No. : CHAENG/2013/51628

scattering phase shifts (SPS) [13, 14]. Satchler et al. [15] have used optical potential model and Dohet-Early and Baye [16] utilised the formalism of unitary correlation operator method to study α-nucleon elastic scattering below 18 MeV and found good agreement with experimental data [17]. In the recent past, Laha and group [18-20] proposed a simple phenomenological potential model for α-nucleon elastic scattering using phase function method (PFM).

Buck et al., [21] have argued that for nucleus-nucleon and nucleus-nucleus system, one can consider local potential without the need for resorting to RGM. Further, they conclude that the non-local potential to be predominantly of similar form as that of local potential. They have obtained scattering phase shifts (SPS's) for α —α and α — ^3He system using local Gaussian potential as the model of interaction with reasonable success. We have obtained α — α SPS by PFM using Gaussian potential, for which the model potential have been obtained through solving time independent Schrodinger equation (TISE) using matrix method [22] in tandem with variational Monte-Carlo (VMC) [23]. Unlike α — α system, the n — α system which is the main focus of this paper has spin-orbit interaction to be present, that gives rise to various j-states observed. In order to obtain the SPS, we employ an innovative technique wherein the PFM is directly used in the optimization procedure for obtaining the model parameters that minimise the mean absolute percentage error (MAPE) between simulated and experimental data.

2. Methodology

2.1. Modeling n-α using Gaussian Potential

The local interaction potential is chosen as Gaussian form given by

$$V(r) = -V_0 e^{-\alpha r^2} + V_{\vec{L}.\vec{S}} \tag{1}$$

Where, V_0 is the depth of potential and α is inverse range parameter.

The non-local potential due to spin-orbit interaction is of similar form as that of local potential and is given by [24]

$$\begin{aligned} V_{\vec{L}.\vec{S}} &= \left(\frac{r_0}{\hbar}\right)^2 \frac{1}{r}\frac{d}{dr}\left(-V_0 e^{-\alpha r^2}\right)\left(\vec{L}.\vec{S}\right) \\ &= \left(\frac{r_0}{\hbar}\right)^2 2\alpha V_0 e^{-\alpha r^2}\left(\vec{L}.\vec{S}\right) \end{aligned} \tag{2}$$

with

$$\vec{L}.\vec{S} = \frac{\hbar^2}{2}\left(J(J+1) - L(L+1) - S(S+1)\right) \tag{3}$$

The centrifugal potential is

$$V_{cf} = \frac{l(l+1)\hbar^2}{2\mu r^2} \tag{4}$$

2.2. Phase Function Method (PFM):

The Schrödinger wave equation for a particle undergoing scattering have energy E and orbital angular momentum ℓ is given by

$$\begin{aligned} \frac{\hbar}{2\mu}\frac{d^2u_\ell(k,r)}{dr^2} + \left[k^2 - \ell(\ell+1)/r^2\right]u_\ell(k,r) \\ = V(r)u_\ell(k,r) \end{aligned} \tag{5}$$

Where $k = \sqrt{E/\left(\hbar^2/2\mu\right)}$.

Second order differential equation Eq.5 has been transformed to the first order non-homogeneous differential equation of Riccati type [25, 26] given by

$$\begin{aligned} \delta_\ell'(k,r) = \\ -\frac{V(r)}{k}\left[\cos\left(\delta_\ell(k,r)\right)\hat{j}_\ell(kr)\right. \\ \left. -\sin\left(\delta_\ell(k,r)\right)\hat{\eta}_\ell(kr)\right]^2 \end{aligned} \tag{6}$$

The Bessel function for different ℓ is known by using

$$\hat{j}_\ell(kr) = (-kr)^\ell\left[\frac{1}{(kr)}\frac{d}{d(kr)}\right]^\ell \frac{\sin(kr)}{(kr)} \tag{7}$$

and

$$\hat{\eta}_\ell(kr) = (-kr)^\ell\left[\frac{1}{(kr)}\frac{d}{d(kr)}\right]^\ell \frac{\cos(kr)}{(kr)} \tag{8}$$

Eq.6 is numerically solved using Runge-Kutta 5^{th} order method with initial condition $\delta_\ell(0) = 0$. For $\ell = 0$, the Riccati-Bessel and Riccati-Neumann functions $\hat{j}_0$ and $\hat{\eta}^0$ get simplified as *sin(kr)* and *—cos(kr),* so Eq.6, for $\ell = 0$ becomes

$$\delta_0'(k,r) = -\frac{V(r)}{k}\sin^2\left[kr + \delta_0(k,r)\right] \tag{9}$$

In above equation the function $\delta_0'(k,r)$ was termed "Phase function" by Morse and Allis [27]. The phase function equation for $\ell = 1$, i.e. P-partial wave, is of the form

$$\begin{aligned} \delta_1'(k,r) = -\frac{V(r)}{k}\left[\cos\left(\delta_\ell(k,r)\right)\left(\frac{\sin(kr)}{kr} - \cos(kr)\right)\right. \\ \left. +\sin\left(\delta_\ell(k,r)\right)\left(\frac{\cos(kr)}{kr} + \sin(kr)\right)\right]^2 \end{aligned} \tag{10}$$

and that for $\ell = 2$, i.e. D-partial wave, is of the form

$$\delta_2'(k,r) = -\frac{V(r)}{k}\left[\cos(\delta_2(k,r))\, f(kr) - \sin(\delta_2(k,r))\, g(kr)\right]^2 \quad (11)$$

where f(kr) is

$$f(kr) = \left(\frac{3}{(kr)^2} - 1\right)\sin(kr) - \frac{3}{kr}\cos(kr) \quad (12)$$

and g(kr) is

$$g(kr) = \left(\frac{-3}{(kr)^2} + 1\right)\cos(kr) - \frac{3}{kr}\sin(kr) \quad (13)$$

The significant advantage of PFM method is that the phase-shifts are directly expressed in terms of the potential and have no relation to the wavefunction. By using a suitable optimization technique we optimize the potential parameters to obtain the SPS's which are in good agreement with experimental SPS.

2.3. Partial & Total Cross-section from SPS

Once, SPS are obtained, one can calculate the partial cross-section $\sigma_\ell(E)$ for each orbital angular momentum ℓ by using following formula as [28]

$$\sigma_l(E) = \frac{4\pi(2l+1)}{k^2}\sin^2(\delta_l(E)) \quad (14)$$

where $k = \sqrt{2\mu E/\hbar^2}$ with μ is the reduced mass and $\delta_l(E)$ is the phahe shift for the respective orbital angular momentum ℓ.

The total cross section is given by [5]

$$\sigma_T = \frac{4\pi}{k^2}\sum_l\left\{(l+1)\sin^2(\delta_l^+) + l\sin^2(\delta_l^-)\right\} \quad (15)$$

where the δ_l^+ denotes the SPS for partial waves with total angular momentum $J = l \pm \frac{1}{2}$ states with $l \geq 1$.

3. Results and Discussion:

3.1. Optimization of Model Parameters

The SPS have been determined, using PFM for the Gaussian potential with spin-orbit coupling term, by fitting the potential parameters so as to obtain the best mean absolute percentage error (MAPE)-value defined as

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|\delta_i^{\exp} - \delta_i^{sim}\right|}{\left|\delta_i^{\exp}\right|}\times 100 \quad (16)$$

where $\delta_i^{\exp}$ and δ_i^{sim} are the experimental and obtained phase-shifts. The smaller the resulting value of *MAPE,* the better the match between the two data sets. The potential parameters alongwith *MAPE* values for *s, p* and d-state are tabulated in Table 1.

3.2. SPS and Potentials

The obtained SPS for $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$-states are shown in Fig. 1. It has been observed that the obtained SPS for these states are in good agreement with that of Satchler et al. [15] upto E_{lab} = 18MeV with *MAPE* values 1.04, 2.98 and 1.17 respectively. For convienience, SPS plot for $s_{1/2}$-state is shifted by 180o to avoid overlapping with other SPS plots. The SPS plots for $d_{3/2}$ and $d_{5/2}$ are plotted separately in Fig. 2 to bring out their match with experimental data clearly, because the SPS for these are small as compared to other states. The left plot window is for $d_{3/2}$ state with *MAPE* of 2.58% and right one is for $d_{5/2}$ state with *MAPE* of 1.32% with optimized potential parameters as given in Table I. For neutron energies above 10 MeV, the obtained phase shifts are not matching well and this is the reason for increase in overall MAPE values given in Table 1.

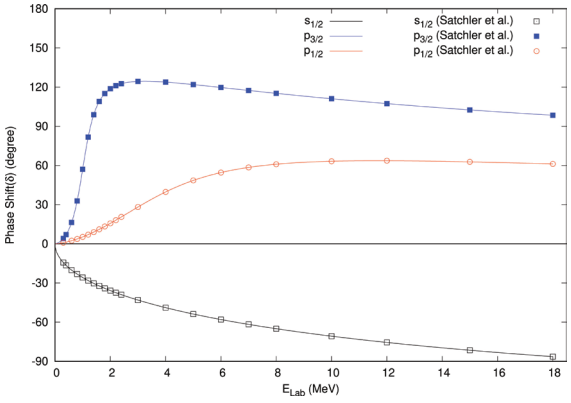

Figure 1: SPS for $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$-state with optimized potential parameters given in Table 1.

Table 1: Model Parameters for Gaussian potential with spin-orbit term.

State	V_0(MeV)	α(fm^{-2})	r_0(fm)	MAPE(%)
$s_{1/2}$	70.71	0.43		1.04
$p_{1/2}$	31.31	0.17	0.37	2.98
$p_{3/2}$	71.69	0.22	0.72	1.17
$d_{3/2}$	6.72	0.17	0.45	2.58
$d_{5/2}$	35.74	0.24	0.39	1.32

In Fig. 3, corresponding potentials for *s, p* and *d* states, obtained using optimized potential parameters given in Table I, are shown. The effect of spin-orbit coupling can be seen clearly in the ordering of potentials. The black curve which is for $s_{1/2}$ is seen to be having maximum depth. This is followed by blue one for $p_{3/2}$, brown for $d_{5/2}$, red for $p_{1/2}$ and finally $d_{3/2}$ in green.

3.3. Partial & Total Cross-Section

Using obtained SPS, both partial cross-section $\sigma_l(E)$ and total cross-section σ_T for *n*-α elastic scattering are calculated for E_{lab} values up to 18 MeV using equation Eq. 14-15. The partial cross-section for $p_{1/2}$ state is shown in Fig.5. The resonance peaks in center of mass energy for $p_{1/2}$ and $p_{3/2}$ are observed at 4.22 MeV and 0.93 MeV and their respective decay widths are 9.3 MeV and 0.91 MeV. It was observed that, for $p_{1/2}$ state, both E_r and Γ values are larger than the values quoted in literature [29-31]. In case of $p_{3/2}$ state both $E_r = 0.93$, $\Gamma = 0.91$ lie within the respective ranges $0.68 < E_r < 0.94$, $0.52 < \Gamma < 1.02$ quoted in literature.

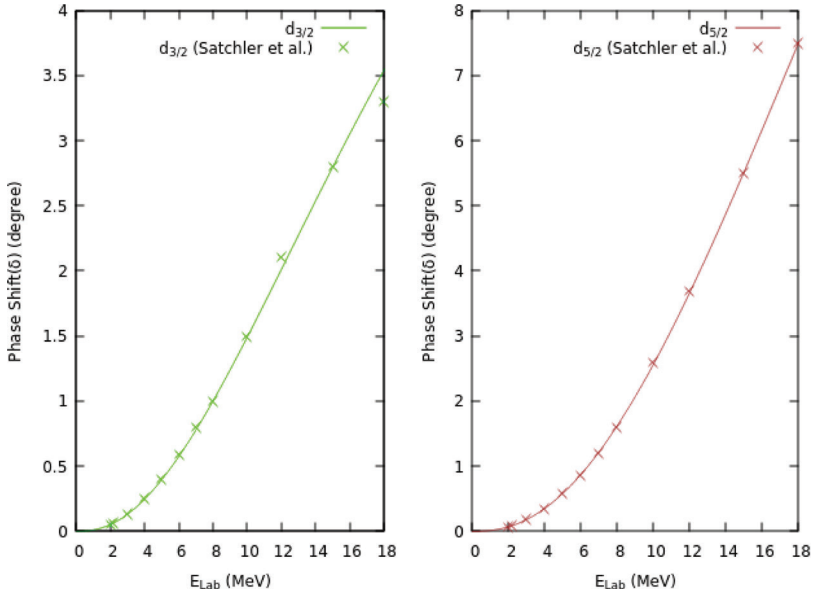

Figure 2: SPS for $d_{3/2}$(left) and $d_{5/2}$-state(right) obtained with optimized potential parameters given in Table 1.

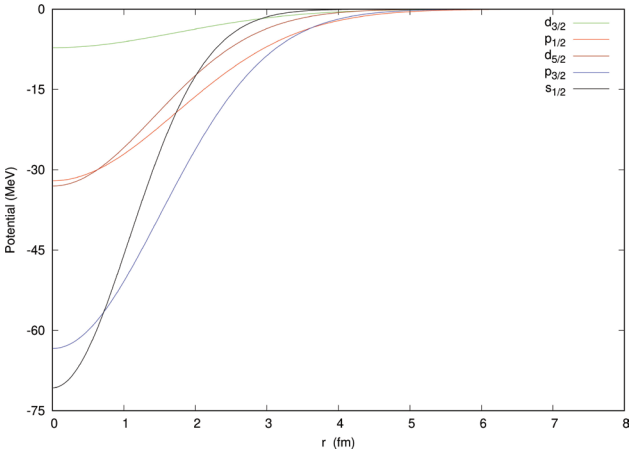

Figure 3: Interaction Potentials for *s*, *.p* and *d*-state with optimized potential parameters given in Table 1.

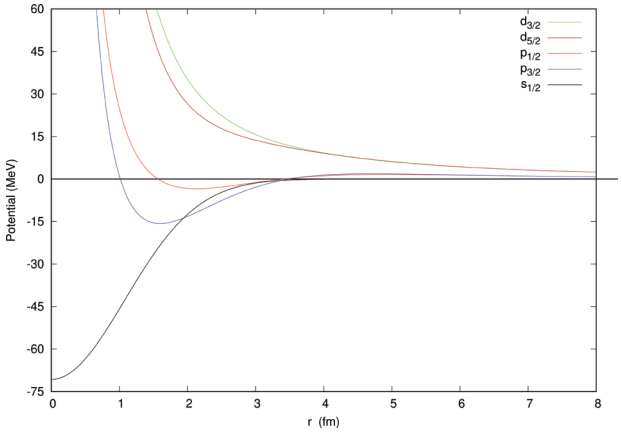

Figure 4: Interaction Potentials for *s*, *p* and *d*-states along with centrifugal term.

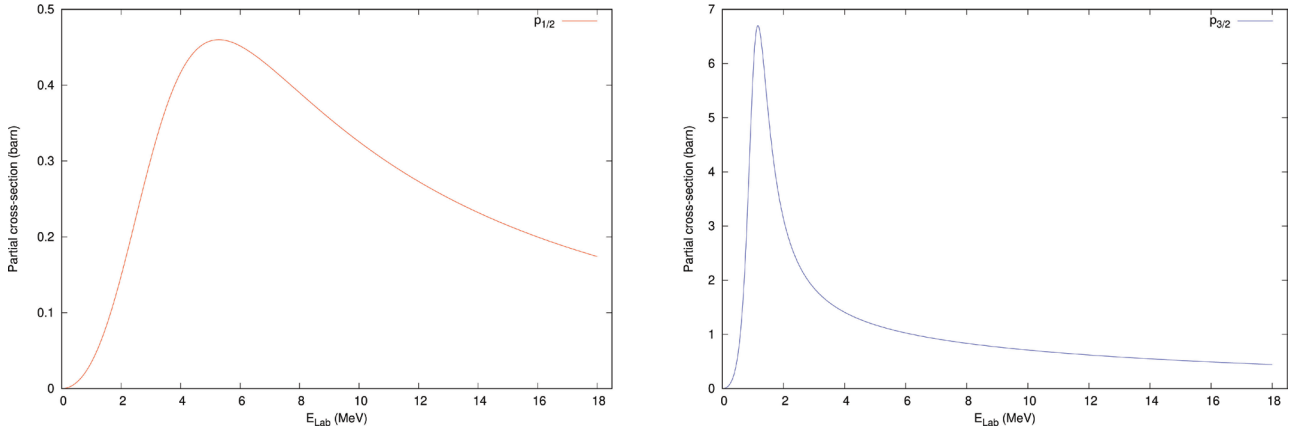

Figure 5: Partial cross-section for $p_{1/2}$(left) and $p_{3/2}$(right)-state.

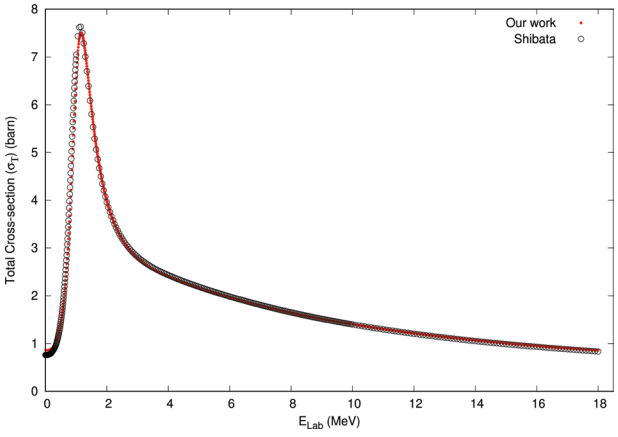

Figure 6: Total cross-section (σ_T) for n-α scattering. Red dots are from our work and empty black circles are from [32].

Conclusion

We conclude that Gaussian potential with spin-orbit term is reasonably good choice of potential for calculating n-α scattering phase shifts using phase function method (PFM). Our computed phase shifts are in good agreement with Satchler et al. In addition, resonance energy E_r and decay width Γ for resonant state $p_{3/2}$ are in agreement with values given in literature but for $p_{1/2}$ state, those values are larger than those given in literature, this is because $p_{3/2}$ is the ground state with sharp resonance about 1MeV energy and the next state $p_{1/2}$ is broad state.

Acknowledgement

We would like to thank the anonymous reviewers for their suggestions and comments for further improvment of this paper.

Authorship Contribution

Lalit Kumar & Anil Khachi: Data curation; formal analysis; investigation; methodology; resources; software; validation; visualization; writing-original draft.
O S K S Sastri: Conceptualization; formal analysis; investigation; methodology; project administration; resources; software; supervision; validation; visualization; writing-review & editing.

Funding

We hereby state that there is no funding recieved for this work.

Conflict of Interest

There is no conflict of interest whatsoever.

Declaration

It is an original data and has neither been sent elsewhere nor published anywhere.

References

[1] T. A. Tombrello and G. C. Phillips, Nucl. Phys., **20**, 648-662 (1960). https://doi.org/10.1016/0029-5582(60)90204-2

[2] A. N. Mitra, V. S. Bhasin and B. S. Bhakar, N. Phys. A and Nucl. Phys. B, **38**, 316-321 (1962). https://doi.org/10.1016/0029-5582(62)91037-4

[3] E. van der Spuy, Nucl. Phys., **8**, 381-414 (1956). https://doi.org/10.1016/S0029-5582(56)80056-4

[4] H. Kanada, S. Naagata, S. Otsuki and Y. Sumi, Prog. Theor. Phys., **30**, 475-493 (1963).

https://doi.org/10.1143/PTP.30.475

[5] P. E. Hodgson, Adv. Phys., **25**, 1-39 (1958). https://doi.org/10.1080/00018735800101157

[6] C. L. Critchfield and D. C. Dodder, Phys. Rev., **76**, 602 (1949). https://doi.org/10.1103/PhysRev.76.602

[7] J. L. Gammel and R. M. Thaler, Phys. Rev., **109,** 2041 (1958). https://doi.org/10.1103/PhysRev.109.2041

[8] L. Brown, W. Haeberli and W. Trächslin, Nucl. Phys. A **90,** 339-352 (1967). https://doi.org/10.1016/0375-9474(67)90238-2

[9] A. Houdayer et al., Phys. Rev. C, **18**, 1985 (1978). https://doi.org/10.1103/PhysRevC.18.1985

[10] J. Berger et al., Phys. Rev. Lett., (1976); Erratum, **37**, 1651 (1976). https://doi.org/10.1103/PhysRevLett.37.1195

[11] L. G. Votta, P. G. Roos, N. S. Chant and R. Woody, Phys. Rev. C, **10,** 520-528 (1974). https://doi.org/10.1103/PhysRevC.10.520

[12] S. Y. Igashov, A. M. Shirokov, Bull. Russ. Acad. Sci. Phys., **71,** 769-775 (2007). https://doi.org/10.3103/S1062873807060044

[13] Y. C. Tang, in Topic in Nuclear Physics II, Lecture Notes in Physics, **145,** 571-692 (1981). https://doi.org/10.1007/BFb0017231

[14] R. Kamouni and D. Baye, Nucl. Phys. A, **791,** 68-83 (2007). https://doi.org/10.1016/j.nuclphysa.2007.04.009

[15] G. R. Satchler, L. W. Owen, A. J. Elwyn, G. L. Morgan, and R. L. Walter, Nucl. Phys. A, **1** (1968). https://doi.org/10.1016/0375-9474(68)90216-9

[16] J. Dohet-Eraly and D. Baye, Phys. Rev. C, **84**, 014604 (2011). https://doi.org/10.1103/PhysRevC.84.014604

[17] G. L. Morgan and R. L. Walter, Phys. Rev., **168**, 1114 (1968). https://doi.org/10.1103/PhysRev.168.1114

[18] U. Laha and J. Bhoi, Phys. Rev. C, **91**, 034614 (2015). https://doi.org/10.1103/PhysRevC.91.034614

[19] J. Bhoi and U. Laha, Brazilian Journal of Physics, **46**, 129-132 (2016). https://doi.org/10.1007/s13538-015-0388-x

[20] A. K. Behera, J. Bhoi, U. Laha and B. Khirali, Commun. Theor. Phys., 72, 075301 (2020). https://doi.org/10.1088/1572-9494/ab8a1a

[21] B. Buck, H. Friedrich, C. Wheatley, Nucl. Phys. A, **275**, 246-268 (1977). https://doi.org/10.1016/0375-9474(77)90287-1

[22] A. Khachi, L. Kumar, A. Sharma and O. S. K. S. Sastri, J. Nucl. Phys. Mat. Sci. Rad. A. 1-5 (2021). https://doi.org/10.15415/jnp.2021.91001

[23] A. Sharma and O. S. K. S. Sastri, Int. J. Quantum Chem., **121**, e26682 (2021). https://doi.org/10.1002/qua.26682

[24] A. Bohr and B. R. Mottelson, Nuclear Structure (World Scientific, Singapore, 1998).

[25] F. Calogero, American Journal of Physics (1968). https://doi.org/10.1119/1.1975005

[26] V. V. Babikov, Sov. Phys. Usp., **10**, 271 (1967). https://doi.org/10.1070/PU1967v010n03ABEH003246

[27] P. M. Morse, W. P. Allis, Phys. Rev., **44**, 269 (1933). https://doi.org/10.1103/PhysRev.44.269

[28] M. Odsuren, K. Katō, M. Aikawa and T. Myo, Phys. Rev. C., **89**, 034322 (2014). https://doi.org/10.1103/PhysRevC.89.034322

[29] A. Csoto and G.M. Hale, Phys. Rev. C, **55**, 536 (1997). https://doi.org/10.1103/PhysRevC.55.536

[30] J. E. Bond and F. W. K. Firk, Nucl. Phys. A, **287**, 317-343 (1977). https://doi.org/10.1016/0375-9474(77)90499-7

[31] A. I. Mazur et al. Phys. Rev. C, **79**, 014610 (2008). https://doi.org/10.1103/PhysRevC.79.014610

[32] K. Shibata, J. Nucl. Sci. Technol., **27**, 81-88 (1990). https://doi.org/10.1080/18811248.1990.9731154

Journal of Nuclear Physics, Material Sciences, Radiation and Applications

Chitkara University, Saraswati Kendra, SCO 160-161, Sector 9-C, Chandigarh, 160009, India

Volume 9, Issue 2 **February 2022** **ISSN 2321-8649**

Copyright: [© 2022 Lalit Kumar, Anil Khachi and O.S.K.S Sastri] This is an Open Access article published in Journal of Nuclear Physics, Material Sciences, Radiation and Applications (J. Nucl. Phy. Mat. Sci. Rad. A.) by Chitkara University Publications. It is published with a Creative Commons Attribution- CC-BY 4.0 International License. This license permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.