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Low Energy S-Wave Proton-Deuteron Scattering Phase-Shifts using Morse Potential

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ABSTRACT

Background: Study of nucleon-nucleus interaction is important to understand the stability of nuclei. At small lab energies ≈ 1-10 MeV, the three body ^{3}He system can be considered as a combination of proton and deuteron two body system. The two body system can be modeled by a local central potential along with Coulomb potential to obtain phase-shifts.

Purpose: Molecular Morse potential has been able to obtain scattering phase shifts of neutron-Deuteron (^{3}H) system successfully [1]. The main objective of this paper is to test if Morse potential proves to be a good interaction potential to study proton-Deuteron (^{3}He) scattering as well.

Methods: The phase function method is solved numerically using Runge-Kutta 5th order (RK-5) method for determining the S-wave scattering phase shifts (SPS) for proton-deuteron (p-D) scattering as a function of proton laboratory energy ranging from 1-10.04 MeV. The model paramters of Morse potential have been varied to obtain best mean absolute percentage error (MAPE) w.r.t. experimental data.

Results: The calculated SPS are found to have MAPE less than 3 percent w.r.t experimental phase shifts [2]. Partial scattering cross-section has been determined using the obtained SPS.

Conclusions: Morse potential has been found to be successful in explaining interaction between proton and deuteron.

1. Introduction

The assumptions of three body interaction has always been a topic of interest for nuclear physicists [3, 4, 5]. And elastic scattering of nucleons with deuterons has been the focus of a considerable theoretical and experimental research in current history [2, 3, 6]. Most of the experimental data for p-D scattering is available for low energies [2, 7, 8, 9]. It is well known that the scattering effects of three nucleon forces are generally trivial at low energies. Below 3 MeV i.e. below the deuteron breakup threshold, many groups have analysed the proton-deuteron (p+d) scattering e.g. Huttel et.al. [7], Kievsky et.al. [9] and Black et.al. [10] obtained the phase shifts for p+d scattering in the proton laboratory energy ranging between 1-3 MeV. S.Ishikawa [11] obtained low energy p-D scattering phase-shifts with a Coulomb-modified Fadeev equation in energy range 0.15-3 MeV. Some groups have been working in the energy range above deuteron breakup threshold but within the elastic scattering range [12]. Clews et.al. [8] did phase shift analysis for p+d scattering in energy region 3-6 MeV, Schmelzbach et.al. [13] did the analysis for 3-5.75 MeV, Christian et.al. [14] studied elastic scattering of proton and neutrons by deuterons in energy region 1-10 MeV and Tornow et.al. [15] did the phase-shift analysis for p+d scattering above the deuteron breakup threshold energies ranging 4-10 MeV. Research has also been undertaken for energies above the elastic scattering range, for p+d scattering. For example Deltuva et.al. [16] compared the Coulomb repulsion between the two protons in the proton laboratory energy region between 3 and 65 MeV and J. Arvieux [2] has given experimental phase shifts for proton laboratory energy ranging from 1-46.3 MeV. Differential and partial scattering cross-section have also been calculated [8, 17, 18] for different energy ranges. In most of these above research papers, [7, 8, 9, 10], the phase-shift data is not available in tabulated format, therefore we are not taking these papers into consideration for our work. In present calculations, we are focussing on laboratory energies of incident protons ranging from E_p = 1 to E_p = 10.04 MeV taken from [2], since above 14 MeV, the inelastic effects come into consideration in proton-deuteron scattering [12, 14, 16]. We do not want to consider any inelasticity in our calculations and would like to work in pure elastic region. In our previous work [1], we have successfully obtained the S-wave phase shifts for three nucleon system ^{3}H in low energy range of neutron laboratory energies from 1-10.5 MeV. Also the maximum value of partial cross-section for p+d scattering is found to be below 10 MeV only. All the aforesaid reasons made us to work in this region.

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After chosing the data for calculations, the next step is to consider the question of how to observe the interaction between the interacting particles. Many interaction potentials like Malfliet-Tjon [19], Manning-Rosen [20], Hulthen [21, 22] have been used to analyse the scattering phase-shifts of scattering caused by neutrons and protons with dueteron. Our group has been working to study nucleon-nucleon and nucleon-nucleus interactions through scattering phase shifts using Morse potential model [23]. In present work, this theoretical model of interaction is applied to study scattering of deuteron nucleus by protons.

2. Simulation Methodology

2.1. Modeling the Interaction using Morse Potential

The main idea behind taking molecular Morse potential as interaction potential to study nucleon-nucleon and nucleon-nucleus interaction is because nucleons themselves experience secondary interactions due to internal quark structure. Binding energy of ^3He is ≈ -7.72 MeV [24] and thus the potential considered to study the nucleon-nucleus interaction should be atrractive in nature. Morse potential also has an attractive nature for low inter nucleon-nucleus (i.e proton and deuteron in current work) distance and fades away when the distance between proton and deuteron increases with an exponentially decaying tail. Hence, the nucleon-nucleus interaction, i.e the interaction between proton and deuteron is modeled using molecular Morse potential [25] given by:

$$V_M(r) = V_0\left[e^{-2\frac{(r-r_m)}{a_m}} - 2e^{-\frac{(r-r_m)}{a_m}}\right] \tag{1}$$

Here, the model parameters V_0, r_m and a_m represents depth or strength of potential, equilibrium distance at which $V = V_0$ and shape parameter respectively.

Since ^3He is a three body system consisting of one neutron and two protons, so at low energies the Coulomb effects can not be neglected and therefore Coulomb potential is also added to complete the interaction. The Coulomb interaction due to protons is given by [26]

$$V_C(r) = z_1 z_2 \frac{e^2}{r} erf(\beta r) \tag{2}$$

where $z_1 z_2 = 2$ for ^3He and β is given as:

$$\beta = \frac{\sqrt{3}}{2 \times R_{pD}} \tag{3}$$

Here, R_{pD} is the root mean square (rms) radius of proton-deuteron (^3He) system taken to be equal to 1.9642 fm [27] thereby giving $\beta = 0.441\ fm^{-1}$. Now the total interaction potential for p-D system is:

$$V(r) = V_0\left[e^{-2\frac{(r-r_m)}{a_m}} - 2e^{-\frac{(r-r_m)}{a_m}}\right] + z_1 z_2 \frac{e^2}{r} erf(\beta r) \tag{4}$$

Or,

$$V(r) = V_0\left[e^{-2\frac{(r-r_m)}{a_m}} - 2e^{-\frac{(r-r_m)}{a_m}}\right] + 2\left(\frac{e^2}{r}\right) erf(0.441r) \tag{5}$$

This interaction potential is put in phase function equation of phase function method (PFM). The main advantage of PFM method is that it only requires interaction potential function to obtain scattering phase shifts (SPS).

2.2. Phase Function Method (PFM)

According to phase function method (PFM), a linear homogeneous equation of second order can be reduced to a first order non-linear differential equation (NDE) by using Green's function approach. The phase equation for Schrödinger equation, has been independently derived by Calogero [28] and Babikov [29] to be in terms of Bessel function $\hat{j}_\ell$ and Riccati-Neumann function $\hat{\eta}_\ell$ as

$$\delta_\ell'(k,r) = -\frac{V(r)}{k}\left[\cos(\delta_\ell(k,r))\,\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\,\hat{\eta}_\ell(kr)\right]^2 \tag{6}$$

Here, k is related to centre-of-mass energy by relation:

$$k_{cm} = \sqrt{\frac{E_{cm}}{\hbar^2 / 2_\mu}} \tag{7}$$

But

$$E_{cm} = \left(\frac{m_D}{m_p + m_D}\right) E_{lab} = 0.667\left(E_{lab}\right) \tag{8}$$

Therefore, in laboratory frame, k will be the function of E_{lab} as:

$$k_{lab} = \sqrt{\frac{2\mu(0.667)E_{lab}}{\hbar^2}} \tag{9}$$

for proton-deuteron system. Equation (6) gives scattering phase-shifts at given lab energy for ℓth partial wave by using initial condition $\delta_\ell(0)=0$. It is evident from equation (6) that the phase shifts δ depend only on the potential V(r) and not on the wave function u(r). For S-wave i.e $\ell=0$, the Bessel function $\hat{j}_\ell$ and Riccati-Neumann function $\hat{\eta}_\ell$ reduces to:

$$\hat{j}_0=\sin(kr) \tag{10}$$

Similarly the Riccati-Neumann function is given as:

$$\hat{\eta}_0=\cos(kr) \tag{11}$$

and thus equation (6) reduces to

$$\delta_0'(k,r)=-\frac{V(r)}{k}\left[\sin\left(kr+\delta_0(r)\right)\right]^2 \tag{12}$$

The non-linear differential equation Equation (12) can be solved using any numerical technique. In this work, we have solved this equation using Runge-Kutta 5^{th} order method (RK-5) to obtain S-wave scattering phase shifts for lab energies of projectile ranging between 1-10.04 MeV.

3. Simulation of Results and Discussion

The scattering phase shifts (SPS) for S-wave have been obtained by solving NDE equation (12) from origin to asymptotic region by RK-5 method. Molecular Morse potential has been taken as the theoretical interaction model to calculate scattering phase shifts for elastic scattering of deuteron by proton. In present study, we have taken proton lab energies less than 10.04 MeV to obtain SPS and compared with available experimental data [2]. The mean absolute percentage error (MAPE) is given by:

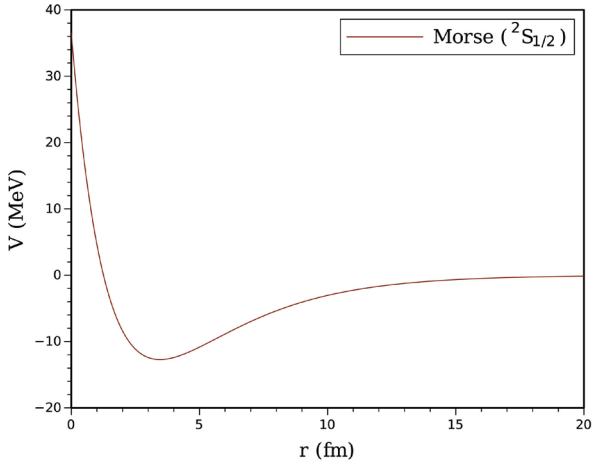
$$NAPE=\frac{1}{N}\sum_{i=1}^{N}\frac{\left|\delta_i^e-\delta_i^o\right|}{\delta_i^e}\times 100 \tag{13}$$

Where δ_i^e and δ_i^o are the experimental and obtained phase-shifts. It is observed that the mean absolute percentage error is less than 3 %. We have obtained the model parameters for Morse potential using Variational Monte Carlo (VMC) technique [30]. The model parameters for Morse potential to obtain S-wave scattering phase shifts have been given in Table 1. Morse potential plot and obtained phase shifts for $^2S_{1/2}$ channel of p-D scattering have been shown

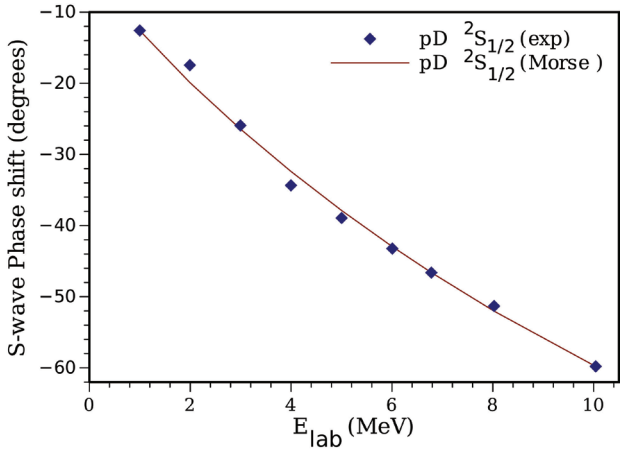
Table 1: Model Parameters of Morse potential to obtain S-wave p-D SPS for E_{lab} (1-10.04)MeV.

State	V_0(MeV)	r_m(fm)	a_m(fm)
$^2S_{1/2}$	12.72	3.46	3.18

in Fig. 1. The potential is clearly an attractive potential with depth $V_0=12.72$ MeV, because of the significant Binding energy of $^3He\approx -7.72$ MeV. The phase shifts have been obtained by integrating phase function equation for S-wave Equation (12) from origin to asymptotic region by using parameters given in Table 1. The simulated S-wave scattering phase shifts for $^2S_{1/2}$ channel of p-D scattering obtained by using



(a) Morse potential



(b) Phase shifts $^2S_{1/2}$ state

Figure 1: (a) Plot of Morse potential and (b) S-wave scttering phase shifts for $^2S_{1/2}$ channel of p-D scattering as function of proton energy in lab frame i.e E_{lab}.

Morse potential along with experimental values [2] are given in Table 2. The relative mean square error (c^2) for the simulated phase shifts in comparison to experimental data [2] has also been calculated and is given as:

$$c^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{\left(\delta_i^e - \delta_i^o\right)^2}{\delta_i^e} \qquad (14)$$

Where δ_i^e and δ_i^o are the experimental and obtained phase-shifts.

Table 2: The scattering phase-shifts for $^2S_{1/2}$ state of p-D system for different lab energies

E_{lab} (MeV)	Exp [2] δ_{Exp}(degree)	Morse(This work) δ_{Exp} (degree)	%error
1.0	-12.6	-12.6	0.1
1.993	-17.5	-19.9	14.0
2.995	-25.9	-26.4	1.9
3.998	-34.4	-32.4	5.7
5.002	-38.9	-37.9	2.7
6.007	-43.2	-43.0	0.6
6.780	-46.6	-46.6	0.0
8.025	-51.3	-52.0	1.4
10.04	-59.8	-59.8	0.0
MAPE (%)			2.91
c^2			0.05

3.1. Cross Section

The partial cross section for ℓth partial wave can be calculated by using formula:

$$\sigma_\ell = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_l \qquad (15)$$

which for S-wave (i.e $\ell = 0$ partial wave), is given by:

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0 \qquad (16)$$

Equation (15) have been used to calculate partial cross section for S-wave using obtained phase-shifts and is given in Fig. 2. No sharp depth is found in our interaction potential which is an indication of the absence of any resonance. Also no peak is found in phase shift and cross section which might infer any non presence of resonance. So, we have ended our calculations at 10.04 MeV which is sufficient for considering elastic effects. Inelasticities do exist at higher energies which have not been taken into account in present work.

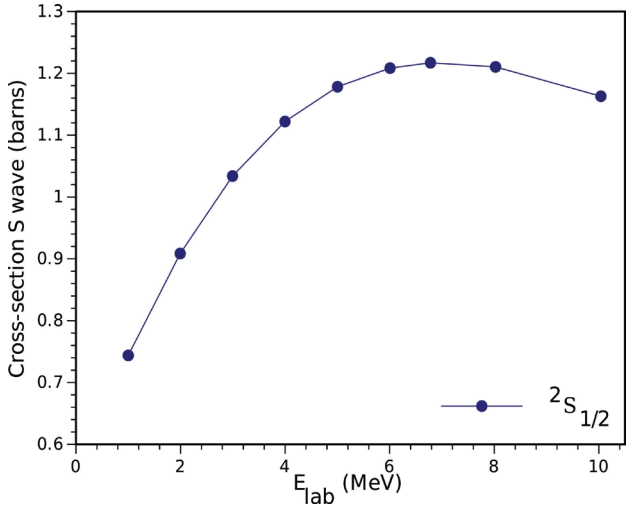


Figure 2: Cross section for $^2S_{1/2}$ channel of p-D scattering as function of proton energy in lab frame i.e E_{lab}.

Conclusion

In present work, we have obtained S-wave scattering phase-shifts for proton-deuteron scattering using Phase function method. The molecular Morse potential has been considered as the theoretical model of interaction. Coulomb potential is added to take care of the interaction between protons. The phase shifts obtained are in good agreement with experimental data [2] with mean absolute percentage error less than 3 % thereby confirming Morse potential to be a good interaction potential in nuclear range as well. We finally calculated S-wave partial cross section for p-D scattering up-to the energies where we have obtained the maximum value of cross-section. It would be interesting to extend this analysis to higher inelastic region and to add doublet and quartet states of p-D which is being attempted.

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Authorship Contribution

Shikha Awasthi and Anil Khachi : Methodology; resources; software; data curation; formal analysis; investigation; validation; visualization; writing original draft.
O. S. K. S. Sastri: Conceptualization; methodology; visualization; formal analysis; investigation; project administration; resources; software; supervision; validation; writing-review editing.

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Conflict of Interest

There is no conflict of interest whatsoever.

References

[1] S. Awasthi, A. Khachi, L. Kumar and O. S. K. S Sastri, J. Nucl. Phys. Mat. Sci. Rad. A **9**, 81 (2021). https://doi.org/10.15415/jnp.2021.91014

[2] Arvieux, J. Nuclear Physics A **221**, 2, 253-268 (1974). https://doi.org/10.1016/0375-9474(74)90317-0

[3] Wu, Y., S. Ishikawa and T. Sasakawa, Few-Body Systems **15**, 4, 145-188 (1993). https://doi.org/10.1007/BF01076246

[4] Aaron, R. and R. D. Amado, Physical Review **150**, 3, 857 (1966). https://doi.org/10.1103/PhysRev.150.857

[5] A. N. Mitra, Nuclear Physics **32**, 529-542 (1962). https://doi.org/10.1016/0029-5582(62)90359-0

[6] C. R. Chen, G. L. Payne, James Lewis Friar and F. G. Benjamin, Physical Review C **39**, 4, 1261 (1989). https://doi.org/10.1103/PhysRevC.39.1261

[7] E. Huttel, W. Arnold, H. Baumgart, H. Berg and G. Clausnitzer, Nuclear Physics A **406**, no. 3, 443-455 (1983). https://doi.org/10.1016/0375-9474(83)90369-X

[8] C. J. Clews and N. Berovic, Physics Letters B **33**, 5, 347-348 (1970). https://doi.org/10.1016/0370-2693(70)90250-9

[9] A. Kievsky, J. L. Friar, G. L. Payne, S. Rosati and M. Viviani, Physical Review C **63**, 6, 064004 (2001). https://doi.org/10.1103/PhysRevC.63.064004

[10] T. C. Black, H. J. Karwowski, E. J. Ludwig, A. Kievsky, Sergio Rosati and M. Viviani, Physics Letters B **471**, 2-3, 103-107 (1999). https://doi.org/10.1016/S0370-2693(99)01366-0

[11] S. Ishikawa, Few-Body Systems **32**, 4, 229-248 (2003). https://doi.org/10.1007/s00601-003-0001-7

[12] C. C. Kim, S. M. Bunch, D. W. Devins, and H. H. Forster, Physics Letters **22**, 3, 314-315 (1966). https://doi.org/10.1016/0031-9163(66)90629-9

[13] P. A. Schmelzbach, W. Grüebler, R. E. White, V. König, R. Risler and P. Marmier, Nuclear Physics A **197**, 1, 273-289 (1972). https://doi.org/10.1016/0375-9474(72)90762-2

[14] R. S. Christian and J. L. Gammel, Physical Review **91**, 1, 100 (1953). https://doi.org/10.1103/PhysRev.91.100

[15] W. Tornow, and H. Witała, Nuclear Physics A **631**, 701-705 (1998). https://doi.org/10.1016/S0375-9474(98)00095-5

[16] A. Deltuva, , A. C. Fonseca, A. Kievsky, S. Rosati, P. U. Sauer and M. Viviani, Physical Review C **71**, 6, 064003 (2005). https://doi.org/10.1103/PhysRevC.71.064003

[17] E. Huttel, W. Arnold, H. Berg, H. H. Krause, J. Ulbricht and G. Clausnitzer, Nuclear Physics A **406**, 3, 435-442 (1983). https://doi.org/10.1016/0375-9474(83)90368-8

[18] C. H. Casella, A. Costantini, B. Lemut, R. Limata, C. Bonetti, L. Broggini and Campajola et al, Nuclear Physics A **706**, 1-2, 203-216 (2002). https://doi.org/10.1016/S0375-9474(02)00749-2

[19] R. A. Malfliet and J. A. Tjon, Nucl. Phys. A **127**, 161 (1969). https://doi.org/10.1016/0375-9474(69)90775-1

[20] B. Khirali, A. K. Behera, J. Bhoi and U. Laha, Ann. Phys. (N. Y) **41**, 168044 (2020). https://doi.org/10.1016/j.aop.2019.168044

[21] J. Bhoi, A. K. Behera and U. Laha, Journal of Mathematical Physics **60**, 8, 083502 (2019). https://doi.org/10.1063/1.5093115

[22] J. Bhoi and U. Laha, Brazilian J. Phys. **46**, 129 (2016). DOI 10.1007/s13538-015-0388-x

[23] A. Khachi, L. Kumar and O.S.K.S Sastri, J. Nucl. Phys. Mat. Sci. Rad. A **9**, 87 (2021). https://doi.org/10.15415/jnp.2021.91015

[24] D. Eyre, A. C. Phillips and Frances Roig, Nuclear Physics A **275**, 1, 13-28 (1977). https://doi.org/10.1016/0375-9474(77)90271-8

[25] P. M. Morse, Phys. Rev. (1929). https://doi.org/10.1103/PhysRev.34.57

[26] S. Ali and A. R. Bodmer, Nuclear Physics **80**, 99 (1966). https://doi.org/10.1016/0029-5582(66)90829-7

[27] Morton, C. Donald, Qixue Wu, and Gordon WF Drake, Physical Review A **73**, 3, 034502, (2006). https://doi.org/10.1103/PhysRevA.73.034502

[28] F. Calogero, American Journal of Physics **36**, 566 (1968). https://doi.org/10.1119/1.1975005

[29] V. V. Babikov, SOV PHYS USPEKHI **10**, 271 (1967). https://doi.org/10.1070/PU1967v010n03ABEH003246

[30] O. S. K. S. Sastri, A. Khachi, L. Kumar, Braz. J. Phys. **52(2)** 1-6 (2022). https://doi.org/10.1007/s13538-022-01063-1

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