

Dependence on the Identification of the Scale Energy Parameter \tilde{Q}^2 in the Quark Distribution Functions for a DIS Production of Z^a

M. Gómez-Bock^{1*}, W. Gonzalez², L. López Lozano³, S. Rosado-Navarro⁴ and A. Rosado⁵

¹Department of AFM, Las Americas University, Santa-Catarina-Martir s/n, San Andrés Cholula, Puebla. México

²Institute of Physics, Meritorious Autonomous University of Puebla (BUAP), PO Box J-48, 52570 Puebla, Mexico.

³Academic area of Mathematics and Physics. Hidalgo State Autonomous University. Kilometer 4.5 Pachuca-Tulancingo highway. 42184 Pachuca, Hidalgo. México.

⁴Faculty of Physical Mathematical Sciences, Meritorious Autonomous University of Puebla (BUAP), PO Box 1364, 72000 Puebla, Mexico.

⁵Institute of Physics, Meritorious Autonomous University of Puebla (BUAP), PO Box J-48, 52570 Puebla, Mexico.

*Email: melina.gomez@udlap.com

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ABSTRACT

We discuss the Z -production in a DIS (Deep Inelastic Scattering) process $e + p \rightarrow e + Z + X$ using the Parton Model, within the context of the Standard Model. In contrast with deep inelastic eP -scattering ($e + p \rightarrow e + X$), where the choice of \tilde{Q}^2 , as the transferred momentum squared, is unambiguous; whereas in the case of boson production, the transferred momentum squared, at quark level, depends on the reaction mechanism (where is the EW interaction taking place). We suggest a proposal based on kinematics of the process considered and the usual criterion for \tilde{Q}^2 , which leads to a simple and practical prescription to calculate Z -production *via* ep -DIS. We also introduce different options in order to perform the convolution of the parton distribution functions (PDFs) and the scattering amplitude of the quark processes. Our aim in this work is to analyze and show how large could be the dependence of the total cross section rates on different possible prescriptions used for the identification of the scale energy parameter \tilde{Q}^2 . We present results for the total cross section as a function of the total energy \sqrt{s} of the system ep , in the range $300 < \sqrt{s} \leq 1300$ GeV.

1. Introduction

The future LHeC (Large Hadron-electron Collider) at CERN will provide the possibility to observe eq collisions with a maximal energy $E^{max} = 60$ GeV of the electron and $E^{max} = 7$ TeV of the proton [1], this means that the maximal total energy of the ep system will be $\sqrt{s} \approx 1300$ GeV. The LHeC is a potentially rich source of Z bosons, will make possible to increase the number of produced Z boson *via* ep collisions up to three orders of magnitude, with respect to previous experiment HERA. In addition, LHeC is considered an excellent collider to search for physics beyond the Standard Model (SM) [2]. However, this can only be accomplished if the results of the SM are well known and established at a high precision level in all its details and without ambiguities. Therefore, we propose in this work a prescription to calculate Z -production unambiguously through the deep inelastic process $e + P \rightarrow e + Z + X$ in the context of the SM and using the Parton Model (PM) approximation [3]. This

approximation together with the theoretical structure of Feynman Diagram formalism for calculations allows us to calculate the ep scattering as fundamental process e -quark scatterings. The calculation is performed using leading order expressions in the QCD-improved Parton Model. [4, 5]. According to the PM, the final step in the evaluation of $d\sigma^{eq}$ consists in consider together the parton cross section $d\sigma^{eq}$ and parton distribution functions (PDFs) $f_q(x, \tilde{Q}^2)$ which represents a probability distribution of the momentum of the quarks in the hadron, and it is a summed over all the possible partons. In the PDFs x ' is the fraction of momentum that the incoming quark carries of the colliding proton and the parameter \tilde{Q}^2 stands for a scale energy, usually identified as the transferred momentum squared in the collisions. At the lowest order in α , the deep inelastic ep scattering ($e + p \rightarrow e + X$), the choice of \tilde{Q}^2 is unambiguous, nevertheless, in the case of Z -production (a five particle process) this does not happens, since the transferred momentum squared depends

on the reaction mechanism, in other words, whether the Z is emitted at the lepton or at the quark line. Our aim in this paper is to show the dependence of the total cross section on the prescription used for the scale parameter \tilde{Q}^2 and the way that one makes the convolution of the PDFs with the amplitude of the quark processes. At the lowest order in α , only two types of reaction mechanism will contribute: Z production at the lepton line and at the quark line at the parton level (see Fig.1). We consider photon as well as Z exchange for the DIS. Diagrams containing the exchange of Higgs boson will be neglected because of the smallness of the Higgs-fermion coupling involved in the mentioned process. We present results for the total cross section as a function of the total energy \sqrt{s} of the system ep , in the range $300 \leq \sqrt{s} \leq 1300 \text{ GeV}$. Taking $\sqrt{s} \approx 1300 \text{ GeV}$ we find that the difference among the rates of the total cross section for the different prescriptions that we have adopted can reach up to

25%. This difference (25%) corresponds to $\approx 10^4$ produced Z boson, at the planned LHeC by taking an integrated luminosity of $0.1 \text{ ab}^{-1}/\text{year}$ [1]. We perform our numerical calculation using the Parton Distribution Functions PDFs reported by Pumplin *et al* [6,7], we use the CTEQ PDFs provided in a $n_f = 5$ active flavors scheme.

We would like to remark that our calculation and analysis for the Z production in a final state $e + Z + X$ via electron-proton deep inelastic interaction, we have taken into account the complete kinematics of the process, for the center of mass energy range of 300 to 1300 GeV , keeping the criterion that the factorization scale parameter \tilde{Q}^2 should be identified with the value of the transferred momentum squared, in the diagrams contributing to the process, then we analyze in detail the dependence of the total cross section on this parameter used for the PM distribution function.

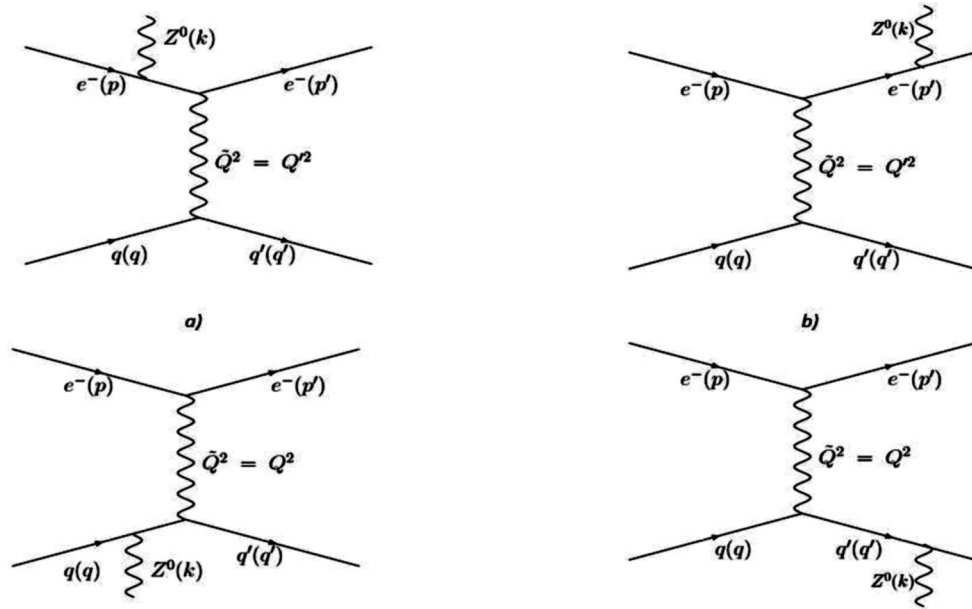


Figure. 1. Feynman diagrams which contribute at the lowest order in α , to the deep inelastic process $ep \rightarrow eZX$, at the quark level, emitted from the initial (a) and final (b) electron, the initial (c) and final (d) quark.

2. Kinematics of the Inclusive Process $e + p \rightarrow e + Z + X$

For the kinematics, we use the formulae presented in Ref. [8], in order to calculate the cross section of the production of a Z boson through the inclusive process

$$e + p \rightarrow e + Z + X \quad (1)$$

We shall denote the four-momenta of these particles by p , P_p , p' and k , respectively. X stands for *anything*. As usual, the following invariants are defined [9]:

$$\begin{aligned} s &= (p + P_p)^2, \\ Q^2 &= -(p - p')^2, \\ v &= P_p \cdot (p - p'), \\ s' &= (p + P_p)^4, \\ Q'^2 &= -(p - p' - k)^2, \\ v' &= P_p \cdot (p - p' - k), \\ W &= -(p + P_p - p' - k)^2, \end{aligned} \quad (2)$$

and the dimensionless variables:

$$x = \frac{Q^2}{2\nu} \quad y = \frac{2\nu}{s} \quad \tau = \frac{s'}{s} \quad x' = \frac{Q'^2}{2\nu'} \quad y' = \frac{2\nu'}{s}$$

The physical region of these kinematical variables has been discussed in detail in Ref. [8] The quark cross section is obtained from the invariant matrix element: $M(eq \rightarrow eqZ)$

$$d\sigma_{tot}^{eq} = \frac{2(2\pi^{-5})}{\zeta} \frac{1}{4} |M_{tot}^{eq}|^2 d\Gamma_3 \quad (3)$$

The 3-particle phase space $d\Gamma_3$ can be expressed with help of the different sets of variables

$$d\Gamma_3 = \frac{\pi s^3}{32} y \frac{dx dy dy' d\tau}{\sqrt{-\Delta_4(p, P_p, p', k)}} \delta(\tilde{Q}^2 - 2\tilde{\nu}') \quad (4)$$

with $\Delta_4(p, q, p', k)$ as Jacobi determinant. The Feynman diagrams which contribute at order in α to M_{tot}^{eq} are depicted in Fig. 2. As we have already said, the Z boson can be produced from the electronic line Fig. 2(a), (b), the quark line Fig. 2(c), (d), then we write

$$M_{tot}^{eq} = M_l^{eq} + M_q^{eq} \quad (5)$$

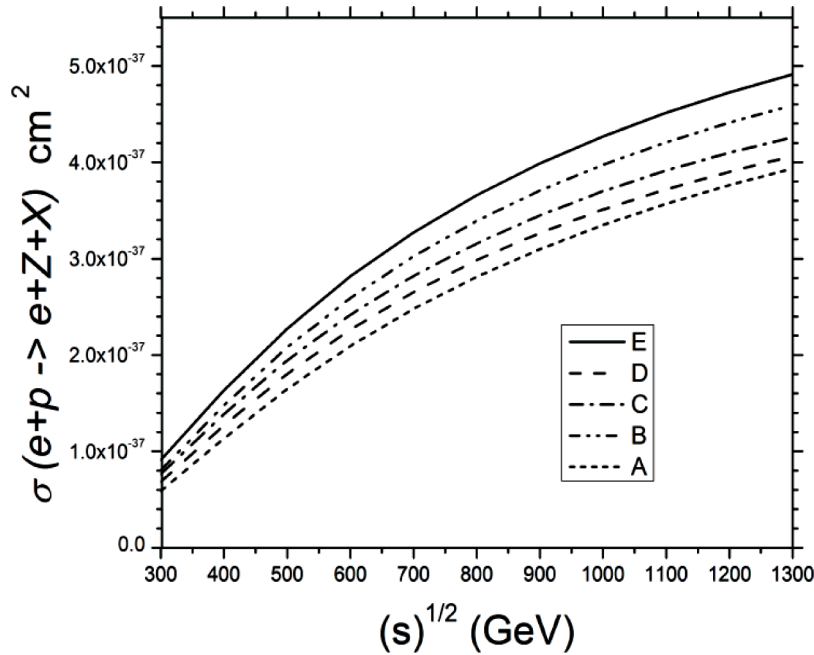


Figure 2. Cross section rates for Z production through the process $e + p \rightarrow e + Z + X$ with a total energy \sqrt{s} in the range $300 \leq \sqrt{s} \leq 1300 \text{ GeV}$ for different options that we have taken to do the convolution of the PDFs with the amplitude of the quark processes (see Eqs. (6)-(10)).

Explicit expressions for the quantities needed for the calculation of $|M_{tot}^{eq}|^2$ are presented in Ref. [8], also the sum of the polarizations of the produced boson is performed there. The final step in the evaluation of $d\sigma_{ep}$ consists now in setting together the parton cross sections $d\sigma_{ep}$ and the parton distribution functions $f_q(x', \tilde{Q}^2)$. In contrast to deep-inelastic ep -scattering where the choice of \tilde{Q}^2 is not ambiguous, in our case it is, since the transferred momentum squared to the nucleon depends on the reaction mechanism (in other words, whether the boson is emitted at the leptonic or at the hadronic line). A detailed investigation of the leading QCD corrections to the simple parton model could give the correct answer. In order to clarify how strong depend the cross section rates on the choice of the scale \tilde{Q}^2 ,

in this work we calculate the following simple prescriptions, some of which are common use.

$$\text{Prescription A} \quad d\sigma^{ep} = \sum_q \left(\int dx' f_q(x', M_Z^2) \cdot d\sigma_{lepton}^{eq} \right. \quad (6) \\ \left. + \int dx' f_q(x', M_Z^2) \cdot d\sigma_{quark}^{eq} \right. \\ \left. + \int dx' f_q(x', M_Z^2) \cdot d\sigma_{inter}^{eq} \right)$$

$$\text{Prescription B} \quad d\sigma^{ep} = \sum_q \left(\int dx' f_q(x', Q^2) \cdot d\sigma_{lepton}^{eq} \right. \quad (7) \\ \left. + \int dx' f_q(x', Q^2) \cdot d\sigma_{quark}^{eq} \right. \\ \left. + \int dx' f_q(x', Q^2) \cdot d\sigma_{inter}^{eq} \right)$$

Prescription C

$$d\sigma^{ep} = \sum_q \left(\int dx' f_q(x', Q^2) \cdot d\sigma_{lepton}^{eq} \right. \\ \left. + \int dx' f_q(x', Q') \cdot d\sigma_{quark}^{eq} \right. \\ \left. + \int dx' f_q(x', Q') \cdot d\sigma_{inter}^{eq} \right) \quad (8)$$

Prescription D

$$d\sigma^{ep} = \sum_q \left(\int dx' f_q(x', (Q^2 + Q'^2)/2) \cdot d\sigma_{lepton}^{eq} \right. \\ \left. + \int dx' f_q(x', (Q^2 + Q'^2)/2) \cdot d\sigma_{quark}^{eq} \right. \\ \left. + \int dx' f_q(x', (Q^2 + Q'^2)/2) \cdot d\sigma_{inter}^{eq} \right) \quad (9)$$

Prescription E

$$d\sigma^{ep} = \sum_q \left(\int dx' f_q(x', Q'^2) \cdot d\sigma_{lepton}^{eq} \right. \\ \left. + \int dx' f_q(x', Q^2) \cdot d\sigma_{quark}^{eq} \right. \\ \left. + \int dx' \sqrt{f_q(x', Q^2)} \sqrt{f_q(x', Q'^2)} \cdot d\sigma_{inter}^{eq} \right) \quad (10)$$

In Eqs. (6)-(10): the first line collects the expressions where the Z boson is emitted from the lepton line ($d\sigma_{lepton}^{ep}$) the second line does the same for the production from the quark line ($d\sigma_{hadronic}^{ep}$) and the last line contains the interference of these two production mechanisms ($d\sigma_{inter}^{ep}$). At this point, we want to clarify how we can understand Prescription E introduced in eq. (10). We introduce Prescription E by making a slight modification in the way that we implement the convolution of the PDFs and the amplitude of the quark sub-processes as follows:

$$d\sigma_{tot}^{ep} = \sum_q \int dx' d\sigma_{tot}^{eq} \quad (11)$$

where

$$d\sigma_{tot}^{eq} = \frac{2(2\pi^{-5})}{\hat{s}} \frac{1}{4} |M_{tot}^{eq}|^2 d\Gamma_3 \quad (12)$$

With

$$|M_{tot}^{eq}|^2 = \left| \sqrt{f_q(x', Q'^2)} \cdot M_l^{eq} + \sqrt{f_q(x', Q^2)} \cdot M_q^{eq} \right|^2 \quad (13)$$

Instead of the usual Parton Model in which one makes the following.

$$d\sigma_{tot}^{ep} = \sum_q \int dx' f_q(x', \tilde{Q}^2) \cdot d\sigma_{tot}^{eq} \quad (14)$$

Where

$$d\sigma_{tot}^{eq} = \frac{2(2\pi^{-5})}{\hat{s}} \frac{1}{4} |M_{tot}^{eq}|^2 d\Gamma_3 \quad (15)$$

And

$$|M_{tot}^{eq}|^2 = |M_l^{eq} + M_q^{eq}|^2 \quad (16)$$

We can observe that prescription E introduces a reasonable scale choice, based on the kinematics of the process $e + p \rightarrow e + Z + X$ and keeping the criterion that the factorization scale parameter should be identified with the transferred momentum squared in the contributing diagram. At the lowest order in α , only the magnitude of the transferred momentum squared of the virtual vector boson exchanged determines whether we are observing a deep inelastic scattering or not. Prescription E is motivated by the fact that the exchange vector boson has to carry enough transferred momentum squared to penetrate deep in the proton structure and this can not be guaranteed if this momentum transfer is taken from the same line where the Z boson is radiated. In other words, when the Z is emitted from the lepton line we have to take $\tilde{Q}^2 = Q'^2$ and when the Z boson is emitted from the quark line then we have to take $\tilde{Q}^2 = Q^2$. For the interference term of the lepton and the quark line mechanism our proposal suggests to take the geometric mean of the PDF associated to each of the two production mechanisms, which contribute to the total amplitude square, $|M_{tot}^{eq}|^2$.

3. Results

Now we present the numerical results of our calculations. We take $M_Z = 91.2 \text{ GeV}$ for the mass of the Z boson and $\sin^2\theta_w = 0.231$ for the electroweak mixing angle [10]. We have included in our computations, besides the photon-exchange, also the Z -exchange diagrams. However, as expected, the dominant contributions to the total cross section come from photon-exchange and specially from leptonic initial state Z emission *i.e.* the diagram depicted in Fig. 1a [8,4]. We give results for the case of unpolarized deep inelastic ep -scattering with a total energy \sqrt{s} in the range $300 \leq \sqrt{s} \leq 1000 \text{ GeV}$ (we remember here that the expected maximal total energy to be reached at the collider LHeC is $\sqrt{s} \approx 100 \text{ GeV}$). We take cuts of 4 GeV^2 , 4 GeV^2 and 10 GeV^2 on Q^2 , Q'^2 and the invariant mass W , respectively. These values are suited for the PDFs reported by Pumplin *et al* [6,7]. We use the CTEQ PDFs provided in $n_f = 5$ active flavors scheme.

There exists already calculation for the contribution to the total cross section $s(ep \rightarrow eZX)$, using different PDFs and

cuts for the momentum transfer squared of the exchanged boson and for the invariant mass W [10,11, 8, 12]. Our results for the total cross section by taking the different prescription given in Eqs. (6)-(10), for $\sqrt{s} = 0 \text{ GeV}$ (HERA) and 1300 GeV (LHeC), are showed in Table I. We show in Fig. 2, the results of the total cross section as a function of the total energy of the ep system, \sqrt{s} we can observe in this plot the evolution of the difference of the results of the cross section rates depending on the prescription used. It is clear that this differences rates of the total cross section can reach up to 25% for the prescription choices of \tilde{Q}^2 or 0 GeV , which is the expected maximal total energy for the system ep to be reached at LHeC.

We found the rates $s(ep \rightarrow eZX)$ is given in Table 1. These values yield to a production of Z -bosons shown in Table 2, and for the different prescriptions, we have chosen to do the convolution of the amplitudes ep subprocesses and the PDFs. We can observe in Table 1, that the differences in the rates of the total cross section can reach up to 25%

for the different choices of \tilde{Q}^2 or the expected maximal total energy for the system ep to be reached at LHeC. This difference (25%) corresponds to a difference of $\sim 10^4$ produced Z bosons, at the planned LHeC, by taking an integrated luminosity of $0.1 \text{ ab}^{-1}/\text{year}$ [1]. By making very simple calculations, we succeeded in showing the importance of trying to find a unique and unambiguous identification of \tilde{Q}^2 or Z -production *via* ep -collisions, in particular for the LHeC. The reason is the following, the energy and luminosity reached at HERA predicted a production of 40-50 Z bosons per year, then the prediction that the ambiguity of \tilde{Q}^2 should give a difference of 11.8% between prescription C and B (See Table 1) was something that was hardly to be seen ($\sim 5-6$). But the same difference (the difference between prescriptions C and B) at LHeC is 4.7%, which implies a ≈ 470 Z bosons of difference between Prescriptions C and B, even than the difference becomes smaller in percentage.

Table 1. Cross section rates in 10^{-37} cm^2 for Z production for $\sqrt{s} = 300 \text{ GeV}$ (HERA) and 1300 GeV (LHeC), for the different prescriptions that we have taken to perform the convolution of the PDFs with the amplitude of the quark processes.

\sqrt{s} (GeV)	$\sigma_A^{ep \rightarrow eZX}$ (10^{-37} cm^2)	$\sigma_B^{ep \rightarrow eZX}$ (10^{-37} cm^2)	$\sigma_C^{ep \rightarrow eZX}$ (10^{-37} cm^2)	$\sigma_D^{ep \rightarrow eZX}$ (10^{-37} cm^2)	$\sigma_E^{ep \rightarrow eZX}$ (10^{-37} cm^2)
300	0.587	0.682	0.763	0.803	0.909
1300	3.934	4.064	4.257	4.590	4.914

Table 2. Number of Z 's that will be produced through the process $ep \rightarrow eZX$ taking $\sqrt{s} = 1300 \text{ GeV}$ (LHeC) and taken an integrated luminosity of $0.1 \text{ ab}^{-1} / \text{year}$ for the different prescriptions that we have taken to perform the convolution of the PDFs with the amplitude of the quark processes.

$\sqrt{s}(\text{GeV})$	$N_A^{ep \rightarrow eZX}$	$N_B^{ep \rightarrow eZX}$	$N_C^{ep \rightarrow eZX}$	$N_D^{ep \rightarrow eZX}$	$N_E^{ep \rightarrow eZX}$
1300	3.934×10^4	4.064×10^4	4.257×10^4	4.590×10^4	4.912×10^4

4. Conclusions

In this work we discussed Z -production at the planned LHeC at CERN, where the expected center of mass energy of $\sqrt{s} \approx 1300 \text{ GeV}$ is 4 times larger than the maximal total energy reached at HERA, $\sqrt{s} \approx 0 \text{ GeV}$. The luminosity planned to be reached at LHeC is at least two orders of magnitude larger than luminosity reached at HERA. We have presented the calculation of the cross section of the Z -production in ep deep inelastic scattering in a range of energies which is expected to be available in the in near future, in the framework of the SM, by using the PM. We showed the dependence of the cross section rates on the scale energy parameter \tilde{Q}^2 . Even than we

can not identify \tilde{Q}^2 unambiguously, we found that is possible to find a reasonable scale choice, based on the kinematics of the process. Our paper can be seen as a first step to look for a possible unique identification of \tilde{Q}^2

Our aim in this paper was to point out that in the case of the production of Z boson or any other particle, we have two transfer square momenta: Q^2 and Q'^2 . In addition, we have demonstrated that although this fact was not important at HERA energies, now could be important at LHeC energies. A detailed comparison of the phenomenological predictions and the experiment results could elucidate on the correct prescription in order to do the convolution of the PDFs and

the amplitudes of the quark processes which leads to the calculation of the $e + p \rightarrow e + Z + X$. However, we can say that our analysis allows to introduce a more clear way to introduce the scale \tilde{Q}^2 . A better way of phrasing this would probably be that one can find from our study a reasonable, practical and simple scale choice, based on the kinematics of the process, the one given by Prescription E. Finally, we end this work making the following comments.

- (a) We can set the dependence of the cross section as a function of the dimensionless variables y , or as a function of dimensionless variables x and y , to use the energy of the final electron and the angle that the momentum of the outgoing electron (p') makes with the momentum of the incident electron (p). In order to define kinematical regions in which the leptonic or the hadronic contribution to the total cross section predominates [8]. In such case, there will be only one momentum transfer square, Q'^2 or Q^2 . Hence, in such regions there will not be more problems with the identification of the scale energy parameter \tilde{Q}^2
- (b) We also want to point out that we did not find any work in which an analysis similar to ours is done, neither to the leading order nor to higher orders. Furthermore, we want to remark here that is difficult to try to extend the analysis done in this work to higher orders, because a box diagram (which certainly there will be box diagrams, which contribute to higher orders) has not a unique momentum transfer, but we can identify two in just one diagram. Hence, it would be necessary to change the criterion that we used in our work: to identify the factorization scale \tilde{Q}^2 is the transferred momentum squared from the diagram which is analyzed.

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