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Radii of Thorium Nuclides Lying in Between the Drip Lines

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ABSTRACT

Background: Nuclear rms radii give information about the nuclear structure, nuclear shape, deformation etc. Microscopic methods are widely used for the study of nuclear structure properties. Hartree-Fock method with an effective interaction of Skyrme force is used for studying the nuclear structure properties.

Purpose: To calculate the rms radii of proton and neutron for thorium nuclei, lying be-tween the drip lines, by using the microscopic mean field theory. The nuclear rms radii data is useful for identifying the shape variation of thorium nuclei, from proton drip line to neu-tron drip line. It also helps to identify the trends in nuclear radii variation as we move to-wards the drip line. This nuclear data will be useful in designing experiments in future and also in understanding the behaviour of complex nuclei. Microscopic study of thorium nuclei is also important in the astrophysical environments.

Methods: This study is based on the Skyrme interacting potential in the Hartree-Fock mean field theory. Iterative diagonalization method with the help of a computational code is used for solving the Hartree-Fock equation.

Results: We have calculated the rms radii of neutron, proton and their total with SV, SLY4 and UDF2 parametrization of the Skyrme force. Neutron rms radii, proton rms radii and total rms radii of thorium nuclei are found to increase with neutron number. UDF2 para-metrization shows an oscillatory nature in the rms radii. SV and SLY4 Skyrme parametrizations are more comparable with other available data. The rms radii variation may be due to the quadrupole deformation, pairing energy and binding energy of the nuclei.

Conclusions: The rms radii of thorium nuclei are found to increase with neutron number. The rms radii obtained by using SV and SLY4 Skyme-parametrizations agree better with the available values.

1. Introduction

Nuclear rms radii give information about the nuclear structure, nuclear shape, deformation etc. Microscopic method is widely used for the study of nuclear structure properties. Hartree-Fock (HF) method with an effective interaction of Skyrme type force is used for studying the nuclear structure properties. This method gives better results for deformed and spherical nuclei [1], as well as neutron deficient and neutron rich nuclei [2]. Different parametrizations like SKM, SLY4, SV, SIII, UDF2 etc. are available for Skyrme force. The accurate information of nuclear rms radii is of special interest because it can explore the asymmetric nuclear density distribution. In recent years, several studies for the determination of nuclear radii were carried out, by using radioactive ion beams [3] and high sensitive laser spectroscopy [4, 5]. Xie et al. [6] had shown the odd-even staggering in nuclear rms radii of light mass nuclei by using laser spectroscopy. Ali A Alzubadi [7] studied the nuclear ground state structure of Zr isotopes by Skyrme Hartree-Fock model with different parametrizations like SKM*, S1, S3, SKM and SKX. He observed that SKX parametrization leads to results having the best agreement with experimental values. Most of the earlier works on rms radii were focused on spherical and light nuclei.

In one of our previous works [8], we have calculated the nuclear level density of thorium nuclei that exist on and off the line of stability. We have noticed some peculiar behaviour in the level density towards the drip line. It may be attribute to the variation of nuclear shape. This prompt us to carry out the in-dept study of nuclear structure of thorium. The thorium nuclei are of special interest because of their various practical applications - they appear in thorium fuelled nuclear reactors and at various stages in nucleosynthesis.

In the present work, we have calculated the rms radii of proton and neutron for thorium nuclei, lying between the drip lines, by using the microscopic mean field theory.

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This study is based on the Skyrme interacting potential in the Hartree-Fock mean field theory. The nuclear rms radii data will be useful in identifying the shape variation of thorium nuclei from proton drip line to neutron drip line. It also helps to identify the trends in the variation of nuclear radii as we move towards the drip line. This nuclear data will be useful in designing experiments in future and also in understanding the behaviour of complex nuclei. Microscopic study of thorium nuclei is also important in the astrophysical environments.

2. Theoretical Formalism

Different theoretical approaches are available for the study of nuclear structure, such as Hartree-Fock (HF) [9], Hartree-Fock-Bogolibov (HFB)[10], relativistic mean field [11, 12], shell model [13, 14], shell correction approach, α-cluster model etc. The nuclear structure studies carried out here is based on the self consistent mean field theory. The Hartree-Fock equation mainly depends on the effective force or nucleon-nucleon interactions used. Cartesian harmonic oscillator (HO) basis with different Skyrme force interaction is used for the simplicity of the solutions. The method using Cartesian coordinate gives information about the nuclear shapes. Iterative diagonalization method, with the help of a computational code, is used for solving the Hartree-Fock equation.

In the Hartree-Fock approximation, the total energy is a functional of non-local density [16],

$$\rho_\alpha\left(r\sigma, r'\sigma\right) = \left\langle \psi' \middle| a^+_{r'\sigma'\alpha} a_{r\sigma\alpha} \psi \right\rangle \tag{1}$$

The local density is the sum of the scalar and vector terms,

$$\rho_\alpha\left(r\sigma, r'\sigma\right) = \frac{1}{2}\rho\alpha\left(r, r'\right)\delta_{\sigma\sigma'} + \frac{1}{2}s_\alpha\left(r, r'\right)\delta_{\sigma\sigma'} \tag{2}$$

The particle density, $\rho_\alpha(\mathbf{r}) = \rho_\alpha(\mathbf{r}, \mathbf{r})$ and $s_\alpha(\mathbf{r}) = s_\alpha(\mathbf{r}, \mathbf{r})$

The Hartree-Fock energy is [15],

$$\varepsilon = \varepsilon^{kin} + \varepsilon^{skyrme} + \varepsilon^{Coul} + \varepsilon^{pair} \tag{3}$$

where ε^{kin}, ε^{Skyrme}, ε^{Coul} and ε^{pair} are kinetic, Skyrme, Coulomb and pairing energies, respectively. The nuclear shape is defined by the surface Σ as

$$\sum : R(\theta,\phi) = e(\alpha)\left[1 + \sum_{\lambda=0}^{\lambda_m ax} \sum_{\mu=\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)\right] \tag{4}$$

In Eqn 4, c(α) **is a function of** $\alpha_{\lambda\mu}$ such that Σ does not depends on α. $\alpha_{\lambda\mu}$ is multipole deformations and $\alpha_{\lambda-\mu} = (-1)^\mu \alpha_{\lambda\mu}$. All multipole deformations are real for $\mu \geq 0$. The length of the principle axis of the volume enclosed by the surface Σ is defined as $R_x = R(\pi/2, 0)$, $R_y = \mathbf{R(\pi/2, \pi/2)}$ **and** $R_z = R(0, 0)$ [16]

Then eigen equation for Routhian is,

$$h'_\alpha \psi_{i,\alpha}(r\sigma) = e'_{i,\alpha}\psi_{i,\alpha}(r\sigma) \tag{5}$$

The i index represent all nucleonic quantum numbers. Hence each single particle state is considered separately and sum over i gives all the neutron and proton single particle states [16]. In HF approach, we expand the single particle wave function $\psi_i(r\sigma)$ into deformed HO wave functions $\psi_{nx\,ny\,nz\,sz}(r\sigma)$ in Cartesian coordinates.

$$\psi_i(r\sigma) = \sum_{n_x=0}^{N_x}\sum_{n_y=0}^{N_y}\sum_{n_z=0}^{N_z}\sum_{s_z=1/2,1/2} A_i^{n_x n_y n_z s_z}\psi_{n_x n_y n_z s_z}(r\sigma) \tag{6}$$

Here N_x, N_y, N_z are the maximum number of HO quanta along x, y and z directions.

Solution of the HF equation with cartesian HO basis depends on the HO frequencies in cartesian directions, ω_x, ω_y, ω_z and number of HO states (M) included in the basis [16]. HO states with the lowest HO single particle energies are,

$$\begin{aligned}\varepsilon_{n_x,n_y,n_z} &= \hbar w_x\left(n_x + \frac{1}{2}\right)\\ &+ \hbar w_y\left(n_y + \frac{1}{2}\right) + \hbar w_z\left(n_z + \frac{1}{2}\right)\end{aligned} \tag{7}$$

with n_x, n_y and n_z being less than N_0 (HO quanta). Both M and N_0 have to be specified to define the basis. For large N_0, the value of M does not depend on N_0 [16]. In the present study, we have considered 15 HO quanta in three directions.

3. Results and Discussion

Nuclear rms radii of proton and neutron were calculated by applying the mean field theory. Deformed harmonic oscillator basis is used for the calculation of rms radii of thorium nuclei. The iterative diagonalization of Hartree-Fock equation gives the solutions with the help of the computational code HFODD. Different parametrization like SV, SLY4 and UDF2 were used for the Skyrme force. Each parametrization gives a different value for radii. We have compared the calculated data with the available values of Kumar et al. [17].

The rms radii is,

$$r = \left[\frac{\int r^2\rho_\alpha(r)d^3r}{\rho_\alpha(r)d^3r}\right]^{\frac{1}{2}} \tag{8}$$

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For rms radii of neutron, ρ_α represents the density of neutron ($\alpha = n$) and for proton rms radii, ρ_α represents the density of proton ($\alpha = p$).

We have calculated the neutron rms radii and proton rms radii of thorium nuclei in the mass range 204-250, by using the HF mean field theory, with different parametrizations in the Skyrme force. Figure 1 shows the rms radii for neutron (left) and proton (right) with the three parametrizations and the comparison with other available values.

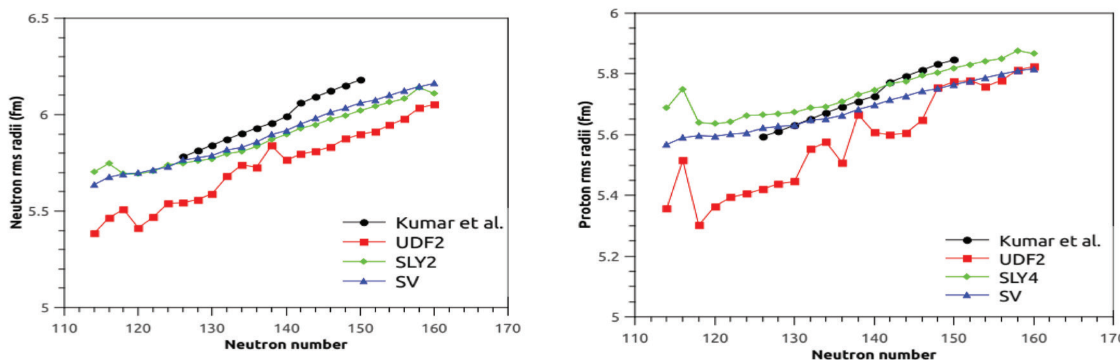


Figure 1: Neutron rms radii (left) and proton rms radii (right) of thorium isotopes with mass number ranging from 204-250 (the corresponding neutron number ranging between 114-160).

The total rms radii of thorium isotopes with neutron number are plotted in Figure 2.

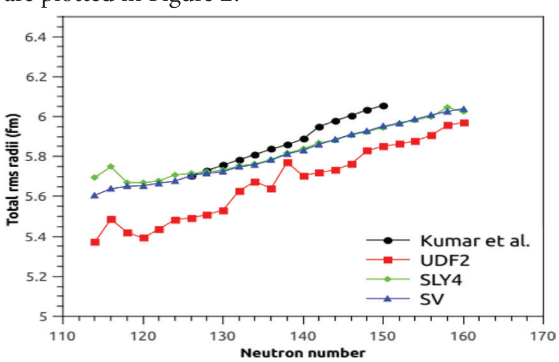
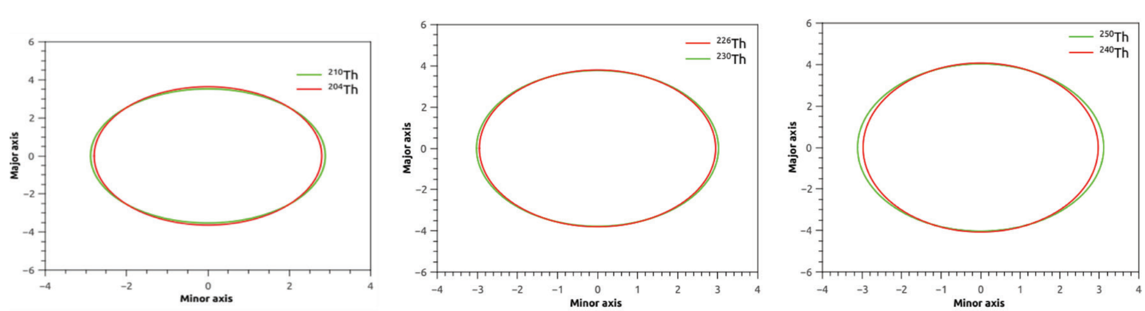


Figure 2: Total rms radii of thorium isotopes with mass number ranging from 204-250 (corresponding neutron number ranging between 114-160).

Each parametrization gives different values of nuclear rms radii. Neutron rms radii, proton rms radii and total rms radii of thorium nuclei are found to increase with the neutron number. We compared our estimated data obtained by using different parametrizations with other available values [17]. The rms radii obtained by using SV and SLY4 Skyme-parametrizations agree better with the available values. UDF2 parametrization shows an oscillatory nature for the rms radii. Nuclear shapes in terms of major and minor axis of the ellipsoid may change as we move away form the β-stability line. The rms radii variation may be due to the quadrupole deformation, pairing energy and binding energy of the nuclei. The rms radii of ^{228}Th (corresponding N=138) comes closer to the compared value while choosing the UDF2 parametrization of the Skyrme force. The observed kink in the rms radii may be due the shape change from one to the other. A sharp increase of rms radii is shown by nuclei having neutron number in the range 114-120 and 136-140 (mass range 204-210 and 226-230).

Nuclear shape variation in terms of major and minor axis of the ellipsoid is also plotted and is shown in Figure 3.



(b) (a) (c)

Figure 3: Shape variation of thorium nuclei with major and minor axes.

From this figure, it is clear that the nuclear deformation is varies from one to other. Nuclei are more deformed towards the neutron drip line. The major axis of Figure 3–(c) is larger than the major axis of Figure 3-(a). The nuclear deformation is varying as we move away from the β-stability line. The variation in rms radii may be due to this deformation.

Conclusion

The rms radii of thorium nuclei in-between the drip lines are calculated by using the Hartree-Fock mean field theory. SV, SLY4 and UDF2 Skyrme interacting potentials have been used for the estimation of radii. We have compared our results from different Skyrme parametrizations with the available data. Rms radii of thorium nuclei is found to increase with neutron number, as we move towards the neutron drip line. The rms radii variation may be due to the quadrupole deformation, pairing energy and binding energy of the nuclei. Nuclear shape variation (in terms of major and minor axis of the ellipsoid) of thorium nuclei towards the drip line is also plotted. The nuclear deformation is varying as we move away from the β-stability line.

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Authorship Contribution

Ummukulsu E involved the workdone and writing of manuscript, Antony Joseph guided the work and provided supervision.

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Conflict of Interest

No

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