



## Jung's Theorem Applied in Nuclear Track Methodology

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### ABSTRACT

Nuclear track density provides accelerator beam imaging and diagnostic employing CR-39 passive detectors. Counting charged particles related tracks by automated reading systems depend on the accuracy of microscope field view other than chemical etching procedure and frequency of overlapped tracks. The study, to propose a method to determine track density for analyser optical field view not calibrated. The approach Jung's theorem, provides the area value based on the maximum distance for two selected etched tracks. Results show that the new method has its importance when microscope field view calibration is not available with precision for accelerator beam diagnostics.

## 1. Introduction

In accelerator beam monitoring employed diagnostic technique by both destructive and non-destructive procedure to determine beam intensity, its longitudinal, transverse shape and size including emittance, *perturbative* or slightly disturbing and destructive method are employed such as scintillator screen for beam intensity study, to determine beam transverse size, shape and position respect the centre line. An alternative plastic detector is particularly useful for low-beam-current (picoAmp) profile monitoring and are successfully employed in accelerator design development. Passive track detector such as  $C_{12}H_{18}O_7$  polymer etched in 6N KOH solution at 60°C for 18h provide images on charged particle as geometrical cones visible under light transmission microscope. The number of tracks per detector area provide track density (tracks/unit area) information for instance on the accelerator beam imaging and diagnostics; the information on beam intensity is derived from the number of particles observed per unit area and the beam tails parameter from track density gradient [1]. It is then essential to have precise value for microscope field value to determine track density or number of tracks per unit area. Polyallyldiglicol carbonate (PADC) currently under the trade mark CR-39<sup>TM</sup> play an important role in determining beam parameter for radioisotope production, personal dosimetry [2] and cancer treatment [3] among other fields. Since track

density provide information on dose transference efficiency i.e. how uniform is both the beam and its spatial dispersion (beam broadening and anisotropy) inside a give volume.

To establish a technique to determine with less uncertainty track density, the Jung theorem is applied to determine the best area in that etched nuclear tracks density shall be determined. The process for track visualization employ a thermoset polymer, PADC detector of 1 mm thick (density: 1.30 g cm<sup>-3</sup>, called Lantrack<sup>TM</sup>, produced by Landauer Inc). The chemical etching process is standardized using a thermostatic water bath in which a 500ml beaker with 6N KOH solution in a thermoregulated water bath at 60±0.1°C Then they were washed with distilled water and dried with absorbent paper, avoiding any mechanical damage to the detectors surface, following a very well establish protocol. Later, the detectors were analyzed with an automatic Digital Image Analysis System (DIAS); detailed description is given by Gammage and Espinosa [4]. Etched tracks are analyzed and displayed by Microsoft Excel<sup>TM</sup> software on a personal computer for further data handling.

## 2. Theoretical approach

Jung's theorem is applied for a plane nuclear detector, under the assumption that the passive device is filled by a given

number of point like etched tracks of small area and negligible overlapping. The theorem provides the area value for a circle of radius  $r$  enclosing all points given by the general expression:

$$r \leq d \sqrt{\frac{n}{2(n+1)}} \quad [1]$$

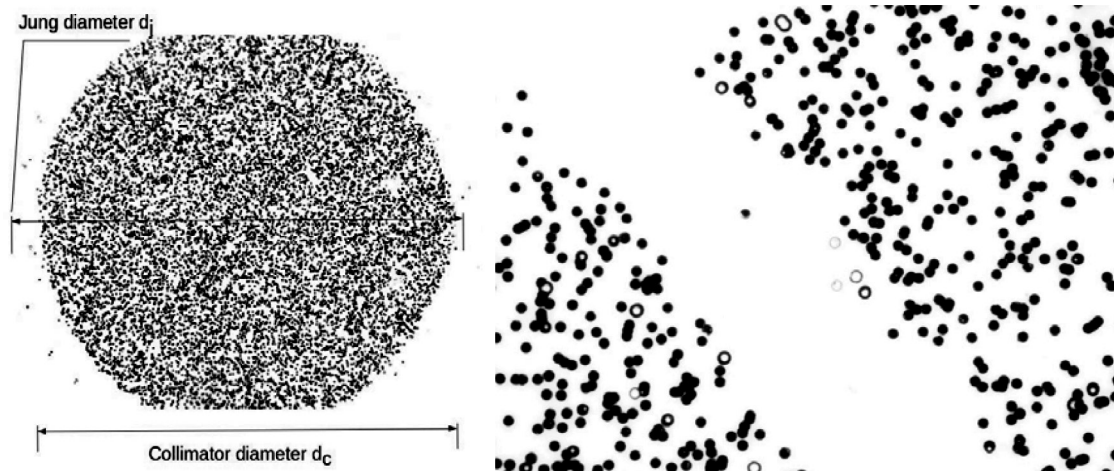
in that  $d$  is the largest Euclidean distance between any two selected tracks. The parameter  $n$  in the case a bi-dimensional space, is  $n = 2$ . The above expression [1] is then re-written in the following form:

$$r \leq d / \sqrt{3} \quad [2]$$

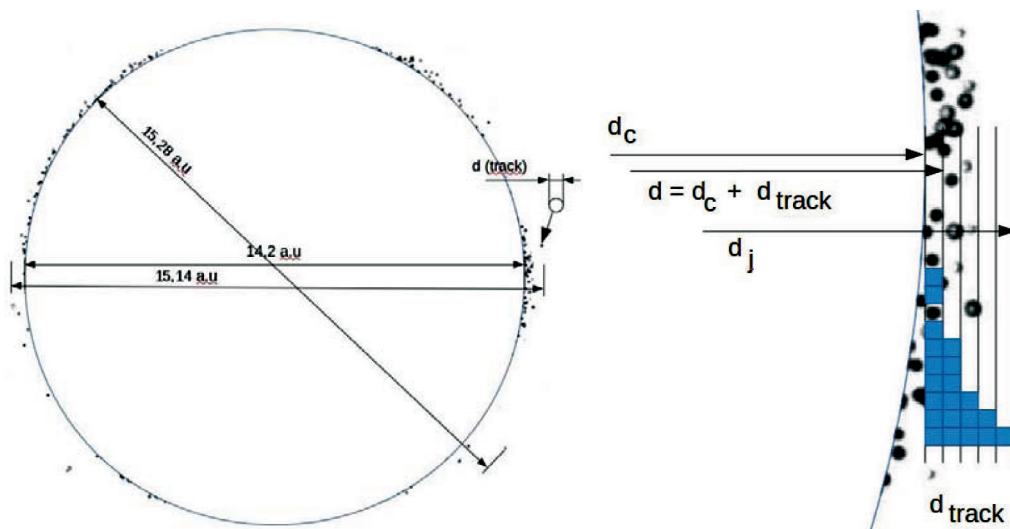
In our case the equality applies for any arbitrary  $n$  number of points or etched tracks distributed randomly and contained in an enveloping circle of radius  $r$ . The Jung theorem is applied to tracks shown in figure 1. The etched track digitalized image, is related to a collimated alpha particle beam registered by the PADC detector. The Jung and the collimator diameter are  $d_j$  and  $d_c$  respectively. In this case we define two areas: the first is related to those tracks that lie in an area defined by the Jung theorem i.e.  $A_j = (\pi / 3) (d_j/2)^2$  where  $d_j$  value is given by the maximum distance between two selected points (tracks) shown in figure 1 on the left as  $d_j = 15.28$  in arbitrary unit (a.u.). the other is the beam collimator diameter  $d_c$ . As it can be observed the two circumferences defining areas do not overlap, the difference contains scattered track. Between the  $d_j$  and  $d_c$  diameter defined area several tracks can be found; these are related to scattering phenomena. Some tracks are induced by alpha particles registered outside the area defined by the collimator that is smaller in comparison to Jung defined area. Since that is determined by the selection of two most distant

tracks, i.e. most probably will be scatter spots, it is convenient to apply some track selection criterium since also these give useful information on the beam characteristics specially for radioisotope production. We suggest therefore to apply the average number of scattered tracks that are registered inside a differential disk given by radius value  $r = r_j + dr$ . The set is related to concentric circle with  $d_c$  increased by discrete values defined let say by track size  $d_c + d_{\text{track}}$  the latter define the first group of scattered tracks, then  $d_c + 2d_{\text{track}}$  the second circle and so on up to  $d_c + nd_{\text{track}}$  in our example  $n = 5$ . In figure 2 at the left the collimator defined circle shadowing the beam size is shown; tracks on the outskirts are the scattered one and as mentioned above are collected in bins having the size of a single-track diameter. On the right side of figure 2, a histogram is depicted. That represents the track distribution per  $n$  indicating the incremental values. Track number diminishes as  $n$  increases.

Tracks outside the collimator circle are randomly distributed and their number follow the Poisson curve. Tracks distribution given by the bar histogram provide the average value  $\langle n \rangle$  and standard deviation  $\sigma$  is determined. As mentioned the two most distant tracks criterion may lead to an incorrect area value. Certainly, some degree of departure from the expected enveloping circle exists due to Coulomb effect however that is expected to be negligible ( $<3\%$ ) in comparison to track density ( $\sim 12-15\%$ ). From the bar histogram  $d'_j$  value the new Jung distance is defined by the expression:  $d'_j = d_c + 2.35 \sigma$ . or  $d'_j = d_c + 2.35 \sqrt{N}$ ; we suggest employing this method to select the diameter value as mentioned to reduce uncertainty in the area estimation corresponding to the actual size of the beam.



**Figure 1.** Digitalized image of etched alpha track pits ( $d_{\text{track}} = 28$  mm) viewed under microscope with two different amplification point like assembly to which Jung’s theorem is applied. On the right, well defined tracks are given; several of them overlap nevertheless they are well distinguished to be counted properly. The apparent white channel is related to two adjacent collimated beams to evidence scattered alpha related tracks.



**Figure 2.** Circle defines the collimator diameter and successive circle contain scattered tracks. On the right, the bar histogram of number of track found outside the collimator per interval having a size of one track diameter.

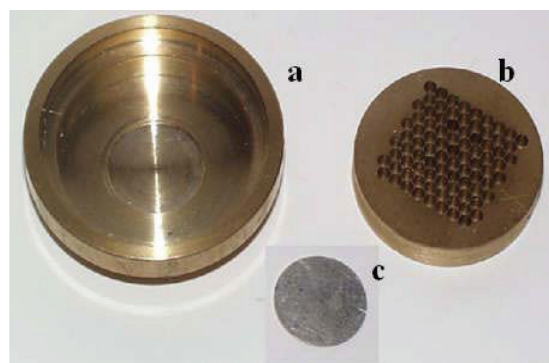
### 3. Experimental Procedure

PADC detector was exposed to  $^{210}\text{Po}$  collimated alpha particle beam ( $E_\alpha = 5.3 \text{ MeV}$ ) for few minutes, then treated chemically to enlarge the damaged region. The track density as mentioned was determined analysing every digitalized track that filled the recognition pattern. The track analyser is based on a software called Morfol described in detail elsewhere and here we mention only that several options exist to be included in a macro program providing a facility to recognize only those track with shape that are selected by the operator. During the process tracks parameters and track pictures are classified by area shape and frequency for further study if required.

#### 3.1. Charged Particle Source Characteristics

Alpha source of  $^{210}\text{Po}$  ( $5 \text{ mCi} = 185 \text{ MBq}$  sealed source model P-2024 with a specific activity of  $166 \text{ TBq.g}^{-1}$ ) is available commercially for anti-static applications from NucleSpot (Staticmaster™ Alpha Ionizer, 2937 Alt Boulevard, PO Box 310, Grand Island, NY 14072-0310). Due to international restrictions on radiation material export, in this experiment it was conveniently employed a homemade, collimated  $^{210}\text{Po}$  alpha source manufactured at the Budapest Research Reactor (Hungary). For this purpose, high purity  $^{209}\text{Bi}$  was exposing to neutrons to induce  $^{209}\text{Bi}(n,g)^{210}\text{Bi}$  reaction. The resulting radioisotope decays (emitting  $\beta$  particles) to  $^{210}\text{Po}$  (half-life  $t \sim 5\text{-day}$ ). Few months build-up time was necessary to have sufficient amount to produce a that the  $^{210}\text{Po}$  electroless deposited on a silver plate and covered by a collimator [5]. The source ( $t \sim 138.4 \text{ days}$ ) produce a collimated beam through a set

of machined holes with a 2mm diameter drilled in a brass plate 6.2 mm thick. This arrangement provides a parallel beam of alpha particles (partially attenuated by air) with 4.60 MeV kinetic energy and the possibility to change at micrometric scale the source-to-detector distance. In figure 3 the source main parts are given for illustration [6]. The source assembly has a silver 15 mm diameter disc (c), placed in a cylinder holder (a) then covered by the collimator (b); both mechanical parts (a and b) are of brass. Further detail on the source specification can be found [6].

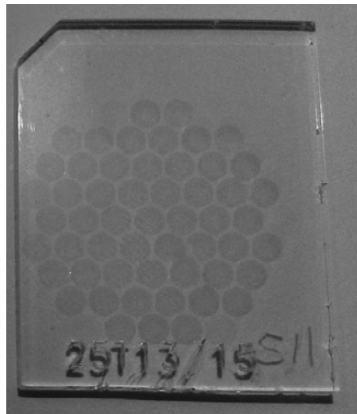


**Figure 3.** Disassembled source: holder (a) and collimator (b). The circular silver plate 15 mm in diameter shown as disks (c)

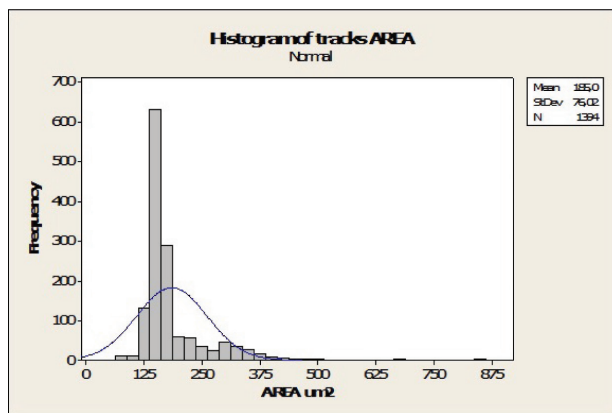
#### 3.2. Collimated Alpha Beam and Etched Track Relationship

Plastic detector after exposure to the alfa source and chemically etched did show clearly the damaged area reflecting the collimator geometrical pattern as reported

in the following Fig. 4. Track diameters were measured to determine their average (28mm) and standard deviation (1.5mm). The resulting histogram showing the Gauss fit for the collimated 4.6 MeV alpha particle track diameter (for 15h etching time), is given in Fig. 5



**Figure 4.** Etched passive detector (2 × 2 cm<sup>2</sup>) evidencing the collimator 56 circular shape holes having an aperture of 2mm in diameter.



**Figure 5.** Etched alpha track pit area (microm<sup>2</sup>) distribution obtained from digitalized images of monoenergetic alphas of 4.65 MeV.

#### 4. Results

Track density related to particle beams, was determined employing a microscope model ZEISS coupled to a canon EOS 60D. Etched tracks were analysed employing the software. Detectors were scanned obtaining 380 digitalized images on average for each particle beam. Images with a resolution of 5184 × 3456 pixels were stored in format .jpg. The microscope field view available in (mm<sup>2</sup>) is 1400 × 1100 (for 5X magnification) and 720 × 530 (10X magnification) corresponding to 1.54 × 10<sup>6</sup> mm<sup>2</sup> and 0.38 × 10<sup>6</sup> mm<sup>2</sup>.

Image processing algorithm through a macro was employed to measure each track pit size through conversion to binary format (white or black). The macros include option of “Make Binary” and “Fill Holes” employed to improve some irregular or incomplete track pit shape. The system provides the track density value form a large set of tracks, the resulting value will be employed for comparison with the value obtained applying the Jung’s theorem for both values i.e. using only the  $d_j$  and the  $d'_j$ .

#### 5. Discussion and Conclusions

In this study, a new method based in the Jung theorem, is suggested to determine track density for accelerator beam characterization and for those cases where the light transmission microscope field view, lack proper calibration. That in our experience may occur when in the microscope the so called “C-mounting” is not the required one. At site at first the Jung theorem can be applied to determine the area where etched tracks surface is visible as point like damages with advantage in the case of collimated beam characterization. In relation to the method, two aspects can be considered; the first is that the method offer best results for tracks area close to point like images with low superimposing or distorted geometry and second is that conveniently the selection method for two tracks should consider the effect of charged particle dispersion (source of accelerator beam broadening and anisotropy). The latter i.e.  $d_j$  is determined applying a selection criterion based on a bar histogram given by the number of tracks outside the expected involving circle (collimator defined circle). Expected track density determined employing the software Image J and values employing the Jung theorem in principle are in good agreement.

**Table 1.** Data comparison for track density measurements (area in arbitrary unit au).

Track image	Square area (au) Track density	Jung distance $d_j$ ; track density	Jung distance $d'_j$ ; track density
Fig. 6	17.7 × 14.5; 0.57	6.91; 0.54	11.5; 0.66
Fig. 7	19.5 × 14.5; 2.94	7.60; 2.44	12.95; 2.27
Fig. 8	17.3 × 3,6; 10.84	4.48; 9.37	7.15; 9.83

Data from Table 1 provide information on the limitations inherent to the method; when the expected track density value is e.g. 0.57, the Jung theorem provides for two different radii circles, 0.54 and 0.66. Thought Jung-average by comparison, differ only few % from the expected track density it is convenient to analyse few areas of the detector

to reduce uncertainty in the resulting track density values. The method where it may be applied with advantage in general terms, is in the case of accelerator beam imaging and diagnostic and specially for hadron therapy and radioisotope production i.e. in those cases that requires some well-known beam characteristics.

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