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Evolution of Shapes and Search for Shape Coexistence in Sd-Shell Nuclei

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Abstract A detailed and systematic study has been performed using state dependent Relativistic Mean-Field plus BCS (RMF+BCS) approach to investigate shape evolution for even-even isotopes of Ne, Mg, Si and S. We perform quadrupole constraint calculation using NL3* parameter and look into the variation of binding energy with respect to deformation and find the shape and deformation corresponding to energy minima. We find various isotopes showing shape coexistence and shape transition while moving from proton drip-line to neutron drip-line. These results are compared with Macroscopic-microscopic approach (Mac-Mic) with Nilson Strutinsky (NS) prescription and some other works and are found consistent for these sd-shell nuclei.

Keywords: Relativistic mean-field theory; Macroscopic-microscopic approach (Mac-Mic); Shape-coexistence; Shape transition; sd-shell nuclei.

1. INTRODUCTION

Evolution of shapes in the nuclei has been investigated consistently by various theoretical and experimental techniques since long. The interesting phenomena of shape transitions and shape coexistence in isotonic and isotopic chain takes place with shell structure of single nucleon levels and has been found in various regions of periodic chart. In sd-shell, in paticular, ^{44}S has been predicted with the prolate-spherical shape coexistence with ”intruder configuration”, being the ground state

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with quadrupole deformation [1]. Using the large-amplitude collective dynamics of shape phase transition in the low-lying states of $^{30\text{-}36}$Mg has been investigated by solving the five-dimensional (5D) quadrupole collective Schrodinger equation [2] and the phenomenon of shape coexistent has been identified in excited 0^+ state in ^{32}Mg [3]. Study of shape evolution in $^{28\text{-}42}$Si has been done by Skyrme-Hartree-Fock model with BCS approximation for the pairing channel [4]. Davies *et al*. has also found shape coexistence and triaxial deformation in Ne, Na, Mg and Al isotopes [5]. Various theoretical models have been showing keen interest in searching the shape coexistence in many isotopic as well as isotonic chains. In this paper, we employ Relativistic Mean-Field plus BCS (RMF+BCS) approach [6, 7] to investigate the shape coexistence and transition in light mass nuclei between Z = 10 − 16 with NL3* force parameter [8]. To fortify our results of shape evolution we will compare our results with our another calculation using Macroscopic-microscopic approach (Mac-Mic) with Nilson Strutinsky (NS) prescription [9, 10] which also includes triaxial deformation and with the results of Lalazissis *et al.* [11].

2. RELATIVISTIC MEAN-FIELD MODEL

RMF calculations have been carried out using the model Lagrangian density with nonlinear terms both for the σ and ω mesons as described in detail in Refs. [12, 13, 14].

$$\begin{aligned}
\mathcal{L} = {} & \bar{\psi}\left[l\gamma^{\mu}\partial_{\mu} - M\right]\psi \\
& + \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} - g\sigma\bar{\psi}\sigma\psi \\
& - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \frac{1}{4}c_{3}\left(\omega_{\mu}\omega^{\mu}\right)^{2} - g_{\omega}\bar{\psi}\gamma^{\mu}\psi\omega_{\mu} \\
& - \frac{1}{4}G_{\mu\nu}^{a}G^{a\mu\nu} + \frac{1}{2}m_{\rho}^{2}\rho_{\mu}^{a}\rho^{a\mu} - g_{\rho}\bar{\psi}\gamma_{\mu}\tau^{a}\psi\rho^{\mu a} \\
& - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\gamma_{\mu}\frac{\left(1-\tau_{3}\right)}{2}A^{\mu}\psi,
\end{aligned}$$

where the field tensors *H, G* and *F* for the vector fields are defined by

$$\begin{aligned}
H_{\mu\nu} &= \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu} \\
G_{\mu\nu}^{a} &= \partial_{\mu}\rho_{\nu}^{a} - Int\partial_{\nu}\rho_{\mu}^{a} - 2g_{\rho} \in^{abc} \rho_{\mu}^{b}\rho_{\nu}^{c} \\
F_{\mu\nu} &= \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}
\end{aligned}$$

and other symbols have their usual meaning. The corresponding Dirac equations for nucleons and Klein-Gordon equations for mesons obtained with the mean-field approximation are solved by the expansion method on the widely used axially deformed Harmonic-Oscillator basis [15, 16]. The quadrupole constrained calculations have been performed for all the nuclei considered here in order to obtain their potential energy surfaces (PESs) and determine the corresponding ground-state deformations [15, 17]. For nuclei with odd number of nucleons, a simple blocking method without breaking the time-reversal symmetry is adopted [18, 19]. In the calculations we use for the pairing interaction a delta force, i.e., $V = -V_0\delta(r)$ with the strength $V_0 = 350$ MeV fm^3 which has been used in Refs. [14, 20, 21] for the successful description of drip-line nuclei.

Apart from its simplicity, the applicability and justification of using such a δ-function form of interaction has been discussed in Ref. [22], whereby it has been shown in the context of HFB calculations that the use of a delta force in a finite space simulates the effect of finite range interaction in a phenomenological manner. For further details of these formulations we refer the reader to Refs. [12, 15, 16].

3. RESULTS AND DISCUSSION

We have performed RMF calculations using NL3* [8] parameter for the entire isotopic chain of Ne, Mg, Si and S and obtained the variation of binding energies (= $-E$) with respect to quadrupole deformation parameter β and evaluated the shape and deformation corresponding to energy minima. Evolution of a variety of shapes is observed in all of these nuclei while moving from the neutron deficient to neutron rich side. Few of these nuclei show a single minima at $\beta = 0$ and consequently referred to as the spherical nuclei whereas many nuclei are found either with the dominant oblate minima (β = negative) or prolate minima (β = positive). Interestingly, some of the nuclei are found with the coexisting prolate and oblate minima and hence referred to as the potential candidates of shape coexistence. Fig. 1, shows the potential energy surfaces traced with respect to the deformation parameter β for all the isotopes of Ne, Mg, Si and S. in (a), (b), (c) and (d) respectively. The energies plotted in Fig. 1 are normalized to zero with respect to the lowest values of energy obtained for each isotopes. One can quickly observe from Fig. 1 that many nuclei show two energy minima with two different shapes coexisting in this sd-shell region. ^{24}Ne, ^{26,28,30}Mg, ^{24,26,28,38,40,46}Si and ^{28,38,44,46,48,50,52,54}S *s*how coexisting prolate and oblate shapes with very small energy differences of the order of few keV and hence are predicted to be potential candidate of shape coexistence. Few nuclei

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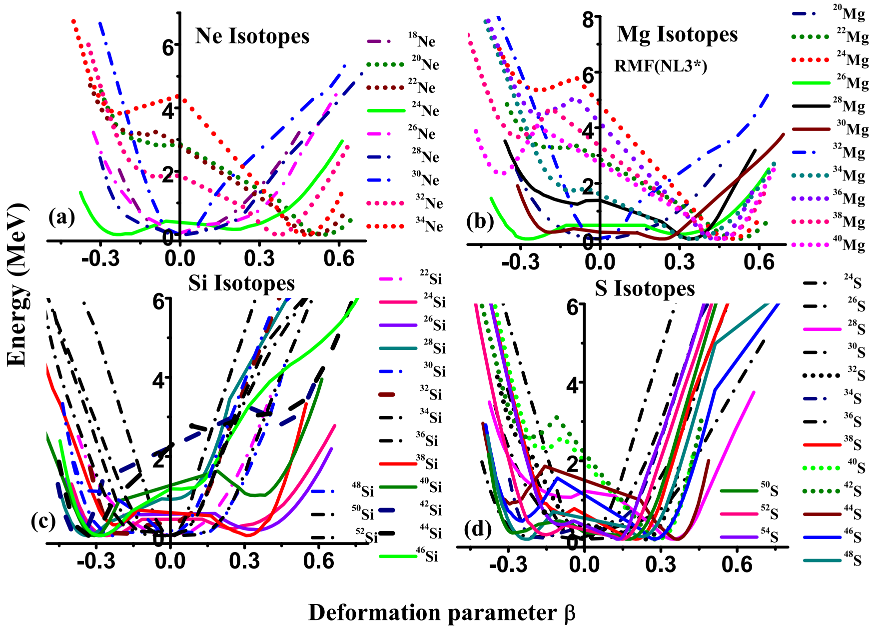

Figure 1: (Colour online) The potential energy surface as a function of the deformation parameter β for Ne, Mg, Si and S isotopes. The dash-dot lines show spherical, short dot show prolate, dash lines show oblate shapes and solid lines show the shape coexisting nuclei.

are found to have single energy minima corresponding to dominant prolate or oblate shapes. The nuclei close to shell closure or with magic neutron number N = 8, 20, 28 i.e. ^{18,26,28,30}Ne, ^{20,32}Mg, ^{22,30,34,36,48,50,52}Si and ^{24,26,30,34,36}S are found spherical in nature as expected.

To further probe the phenomenon of shape coexistence in this region and to find the most probable candidates showing shape coexistence among these nuclei, we have displayed excitation energies which are actually the difference between the energies of prolate and oblate minima in Fig. 2 for nuclei showing two minima together. We observe from the figure that the energy difference is found very small of the order of few keV's for ^{24}Ne, ^{26,30}Mg, ^{24,26,38}Si and $^{38,46-54}$S indicating shape coexistence between prolate and oblate shapes.

We show the evolution of shapes along with the variation of deformation parameter β with neutron number N corresponding to all the isotopes of Ne, Mg, Si and S in Fig. 3. For comparison, we have also shown our calculated values of Mac-Mic approach [9, 10] and the data of Lalazissis *et al*. [11] which show reasonable agreement. Deformation near magic number N = 8, 20, 28 is smaller than the nuclei away from shell closure as expected. The shape transition from prolate

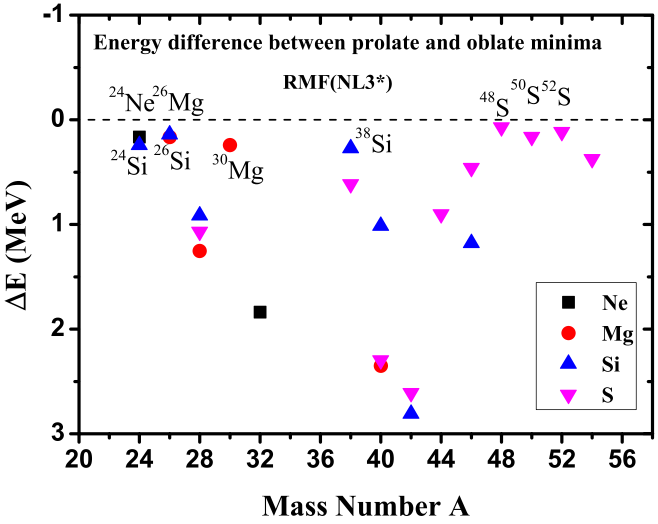

Figure 2: (Colour online) Difference between energy of prolate and oblate minima is plotted for few candidates from Ne, Mg, Si and S isotopes.

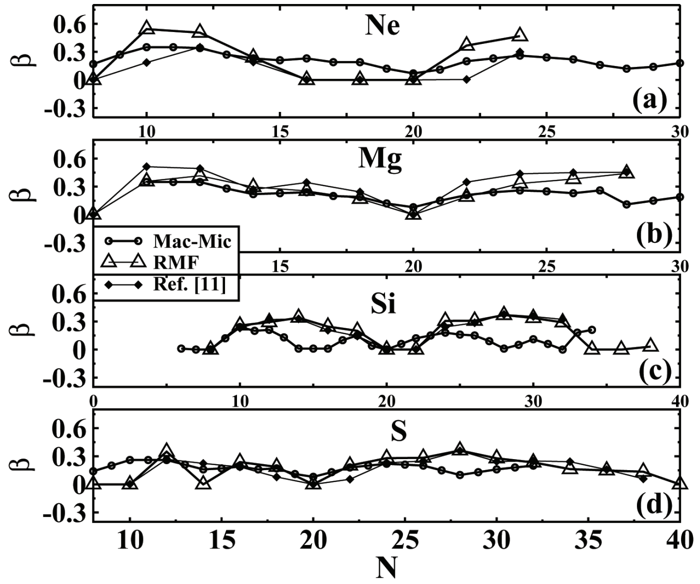

Figure 3: Variation of deformation β with respect to neutron number N for Ne, Mg, Si and S isotopes. For comparison data from Ref [11] are also plotted.

to oblate while moving from ^{38}Si to ^{40}Si and from ^{46}S to ^{48}S is evident in Fig. 3. as predicted by both RMF and Ref. [11]. It may be noted that most of the Ne, Mg and S isotopes show prolate shapes whereas Si isotopes show predominantly oblate shapes in agreement with prediction of Lalazissis *et al*. [11]. However Mac-Mic results show slight deviation in predicted data of deformation and shape due to inclusion of triaxiality. Although the β values show good agreement with our results of RMF and Ref. [11] except at few places where the shapes are traiaxial. Evolution of shapes (with oblate ($\gamma = -180^o$), triaxial ($-120^o < \gamma < -180^o$) and prolate ($\gamma = -120^o$)) using Mac-Mic is shown in Fig. 4, where one can see that the prolate and oblate shapes are dominant shape phases in agreement with our RMF results but few nuclei are predicted to have triaxial shapes in this region as also predicted by Ref. [5].

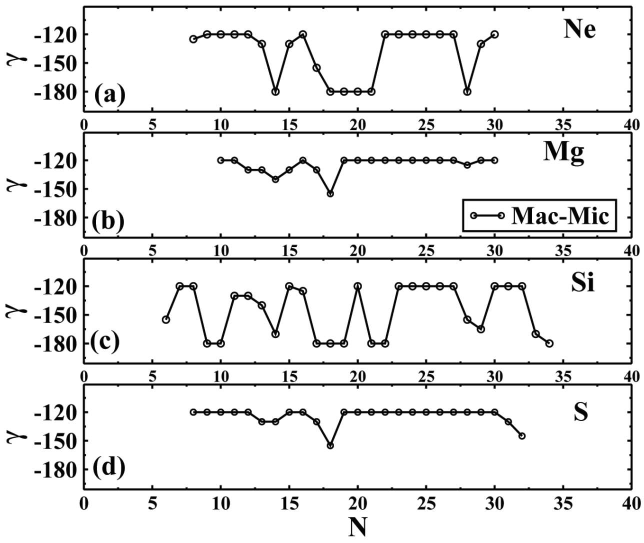

Figure 4: Shape parameter γ Vs neutron number N.

4. SUMMARY

To summarize, we have employed Relativistic Mean-Field plus BCS approach and Macroscopic-microscopic approach to study the systematic behavior of shape evolution in sd-shell nuclei with $Z = 10 - 16$. We have found mixed shapes in this particular region in which some nuclei are found spherical near or at magic neutron number. Prolate shape is the predominant shape phase in this region with few oblate and traixial shapes. Many nuclei show two energy minima corresponding to prolate and oblate shapes indicating shape coexistence. ^{24}Ne, ^{26,30}Mg, ^{24,26,38}Si and $^{38,46-54}$S are predicted to be the potential candidates of shape coexistence. Shape transitions from prolate to oblate and few triaxial are observed in this region. Our calculations show agreement with other theoretical works as well.

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