DOI: 10.15415/jnp.2017.42031

Understanding the Basics of Final Unification With Three Gravitational Constants Associated With Nuclear, Electromagnetic and Gravitational Interactions

U. V. S. SESHAVATHARAM[1] AND S. LAKSHMINARAYANA[2]

[1]Honorary Faculty, I-SERVE, Survey no-42, Hitex road, Hitech city, Hyderabad-84, Telangana, India.

[2]Department of Nuclear Physics, Andhra University, Visakhapatnam-03, AP, India

Email: seshavatharam.uvs@gmail.com

Received: October 18, 2016| Revised: December 14, 2016| Accepted: January 08, 2017

Published online: February 06, 2017

Abstract With three fundamental gravitational constants assumed to be associated with strong interaction, electromagnetic interaction and gravity, we review the basics of final unification.

Keywords: Final unification; Gravitational constants associated with strong and electromagnetic interactions.

1. INTRODUCTION

Even though ‘String theory’ models and “quantum gravity’ models [1, 2, 3] are having a strong mathematical back ground and sound physical basis, they are failing in implementing the Newtonian gravitational constant [4] in atomic and nuclear physics and thus seem to fail in developing a ‘workable’ model of final unification. It clearly indicates our lack of understanding and uncertain assumptions on which our current physics is being built up. The main issue is: to understand the basics of final unification from hidden, unknown and un-identified physics! Based on the old and ignored scientific assumption put forward by Nobel laureate Abdus Salam, we developed and compiled many interesting semi empirical relations assumed to be connected with nuclear physics, atomic physics and astrophysics [5, 6]. Based on ‘workability’, we appeal the readers to go through.

Journal of Nuclear Physics, Material Sciences, Radiation and Applications
Vol-4, No-2, Feburary 2017
pp. 355-373

CHITKARA UNIVERSITY

2. TWO BASIC ASSUMPTIONS OF FINAL UNIFICATION

Assumption-1: Magnitude of the gravitational constant associated with the electromagnetic interaction is, $G_e \cong (2.375 \pm 0.002) \times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

Assumption-2: Magnitude of the gravitational constant associated with the strong interaction is, $G_s \cong (3.328 \pm 0.002) \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

We chose (G_e, G_s) in such a way that,

$$\frac{m_p}{m_e} \cong \left(\frac{G_s m_p^2}{\hbar c}\right)\left(\frac{G_e m_e^2}{\hbar c}\right) \tag{1}$$

$$\left(\frac{G_s m_p m_e}{\hbar c}\right) \cong \left(\frac{\hbar c}{G_e m_e^2}\right) \tag{2}$$

Considering the two pseudo gravitational constants assumed to be associated with strong and electromagnetic interactions,

1) Currently believed generalized physical concepts like, proton-electron mass ratio, neutron life time, weak coupling constant, strong coupling constant, nuclear charge radius, root mean square radius of proton, melting points of proton and electron, nuclear charge radii, nuclear binding energy, nuclear stability, Bohr radius of hydrogen atom, electron and proton magnetic moments, Planck's constant, atomic radii, molar mass constant and Avogadro number etc. can be reviewed in a unified approach and can be simplified.
2) Significance of the ratio of nuclear gravitational constant and Newtonian gravitational constant can be understood and thereby magnitude of the Newtonian gravitational constant can be estimated in a unified approach.
3) Considering the ratio of nuclear gravitational constant and Newtonian gravitational constant, neutron star mass can be understood.

3. TO UNDERSTAND THE ROLE OF NEWTONIAN GRAVITATIONAL CONSTANT IN NUCLEAR PHYSICS

After developing many relations, to a very good accuracy, we noticed that,

$$m_p \cong \left(\frac{G_N}{G_e}\right)^{\frac{1}{6}} \sqrt{M_{pl} m_e} \tag{3}$$

where $M_{pl} \cong \sqrt{\hbar c/G_N}$ is the Planck mass. To proceed further, let,

$$m_{npl} \cong \sqrt{\frac{\hbar c}{G_s}} \approx 546.7 \text{ MeV}/c^2 \cong \text{Nuclear Planck mass} \tag{4}$$

In terms of the nuclear Planck mass,

$$m_p \cong \left(\frac{m_e^6 M_{pl}}{m_{npl}^2}\right)^{\frac{1}{5}} \tag{5}$$

$$m_e \cong \left(\frac{m_p^5 m_{npl}^2}{M_{pl}}\right)^{\frac{1}{6}} \tag{6}$$

In a simplified picture, proton and electron rest masses and reduced Planck's constant can be expressed in the following way.

$$m_e \cong \left(\left(\frac{G_N}{G_s}\right)\left(\frac{\hbar c}{G_s}\right) m_p^{10}\right)^{\frac{1}{12}} \tag{7}$$

$$m_p \cong \left(\left(\frac{G_s}{G_N}\right)\left(\frac{\hbar c}{G_s}\right)^{-1} m_e^{12}\right)^{\frac{1}{10}} \tag{8}$$

$$\hbar \cong \left(\frac{G_s}{G_N}\right)\left(\frac{m_e}{m_p}\right)^{10}\left(\frac{G_s m_e^2}{c}\right) \tag{9}$$

By fixing the magnitude of (G_S), magnitude of (G_N) can be fixed.

4. TO UNDERSTAND THE PLANCK'S CONSTANT AND TO FIX THE MAGNITUDE OF G_S

Proceeding further, it is possible to obtain the following relations.

$$h \cong \sqrt{\frac{m_p}{m_e}} \sqrt{\left(\frac{G_s m_p^2}{c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 c}\right)} \tag{10}$$

$$hc \cong \sqrt{\frac{m_p}{m_e}} \sqrt{\left(G_s m_p^2\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)} \tag{11}$$

Note that, these two relations are free from arbitrary coefficients and seem to be connected with quantum theory of radiation. With further research, if one is able to derive these two relations, unification of quantum theory and gravity can be made practical and successful. Based on these relations,

$$\begin{aligned} G_s &\cong \frac{4\pi\varepsilon_0 h^2 c^2 m_e}{e^2 m_p^3} \\ &\cong 3.329560807 \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned} \tag{12}$$

$$\begin{aligned} G_e &\cong \left(\frac{e^2 m_p^2}{16\pi^3 \varepsilon_0 m_e^4}\right) \\ &\cong 2.374335471 \times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned} \tag{13}$$

$$\begin{aligned} G_N &\cong \left(\frac{m_e}{m_p}\right)^{14} \left(\frac{4\pi\varepsilon_0}{e^2}\right)^2 \left(\frac{2\pi h^3 c^3}{m_p^2}\right) \\ &\cong 6.679856051 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned} \tag{14}$$

5. FITTING AND UNDERSTANDING THE NEUTRON LIFE TIME AND STRONG COUPLING CONSTANT

It may be noted that, during beta-decay, by emitting one electron and one neutrino, neutron transforms to proton.

Let, t_n be the life time of neutron, m_n be the rest mass of neutron and $(m_n - m_p)$ be the mass difference of neutron and proton. Then, quantitatively it is possible to show that,

$$\frac{\left(m_n - m_p\right)}{m_n} \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{G_s m_n}{c^3 t_n}\right) \tag{15}$$

where G_N is the Newtonian gravitational constant. Very interesting observation is that, the three gravitational constants seem to play a simultaneous role in deciding the neutron decay time and is for further analysis. Now,

$$t_n * \left(m_n - m_p\right)c^2 \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{G_s m_n^2}{c}\right) \tag{16}$$

$$t_n \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{G_s m_n^2}{\left(m_n - m_p\right)c^3}\right) \approx 896 \text{ sec} \tag{17}$$

With 1-2 % error, this obtained value can be compared with recommended [7] and experimental neutron life times of (878 to 888) [8]. With reference to the Weak coupling constant G_F and the proposed gravitational constant associated with strong interaction G_S,

$$t_n \cong \left(\frac{m_e}{m_p}\right)^{\frac{7}{2}} \frac{\sqrt{G_s G_F}}{2G_N\left(m_n - m_p\right)c} \tag{18}$$

$$G_N \cong \left(\frac{m_e}{m_p}\right)^{\frac{7}{2}} \frac{\sqrt{G_s G_F}}{2t_n\left(m_n - m_p\right)c} \tag{19}$$

Qualitatively, if one is willing to define the well believed strong coupling constant α_s with the following relation,

$$\alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.11519371 \tag{20}$$

error in estimation of neutron life can be minimized and can be expressed with the following relation.

$$t_n \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{1}{\alpha_s}\right)^{\frac{1}{2}} \left(\frac{\hbar}{\left(m_n - m_p\right)c^2}\right) \cong \frac{303.42 \text{ sec}}{\sqrt{\alpha_s}} \tag{21}$$

With reference to recommended value [7] of $\alpha_s \cong 0.1185 \pm 0.0006$, obtained $t_n \cong 881.422$ sec

$$\alpha_s \cong \left(\frac{G_e}{G_N}\right)\left(\frac{\hbar}{t_n\left(m_n - m_p\right)c^2}\right)^2 \cong \left(\frac{303.42 \text{ sec}}{t_n}\right)^2 \tag{22}$$

With reference to recommended value of $t_n \cong (880.3 \pm 1.1)\text{sec}$, obtained $\alpha_s \cong 0.1188$

6. NUCLEAR CHARGE RADIUS AND ROOT MEAN SQUARE RADIUS OF PROTON

Nuclear charge radius can be expressed with the following relation.

$$R_0 \cong \frac{2G_s m_p}{c^2} \approx 1.24 \times 10^{-15} \text{ m} \tag{23}$$

Considering this relation (23), magnitude of G_N can be estimated with the following relation.

$$G_N \cong \left\{\left(\frac{m_e}{m_p}\right)^{12}\left(\frac{c^3 R_0^2}{4\hbar}\right)\right\} \cong 4.35 \times 10^{19} R_0^2 \tag{24}$$

Root mean square radius of proton [7] can be expressed with the following relation.

$$R_p \cong \frac{\sqrt{2} G_s m_p}{c^2} \approx 0.876 \times 10^{-15} \text{ m} \tag{25}$$

Considering this relation (25), magnitude of G_N can be estimated with the following relation.

$$G_N \cong \left\{\left(\frac{m_e}{m_p}\right)^{12}\left(\frac{c^3 R_p^2}{2\hbar}\right)\right\} \approx 8.7 \times 10^{19} R_p^2 \tag{26}$$

7. TO FIT AND UNDERSTAND THE FERMI'S WEAK COUPLING CONSTANT

To a great surprise, it is noticed that[7],

$$G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \tag{27}$$

From above relations,

$$\begin{aligned} G_F &\cong \left(\frac{m_e}{m_p}\right)^2 \left(\frac{4G_s^2 m_p^2 \hbar}{c^3}\right) \cong \left(\frac{4G_s^2 m_e^2 \hbar}{c^3}\right) \\ &\approx 1.44 \times 10^{-62} \text{ J.m}^3 \end{aligned} \tag{28}$$

Based on this relation (28), magnitude of G_N can be estimated with the following relation.

$$\begin{aligned} G_N &\cong \left(\frac{m_e}{m_p}\right)^{10} \left(\frac{G_F c^2}{4\hbar^2}\right) \\ &\cong 6.66 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{ sec}^{-2} \end{aligned} \tag{29}$$

where, $G_F \cong 1.1663787 \times 10^{-5} (\hbar c)^3 \text{ GeV}^{-2}$
$\cong 1.435850781 \times 10^{-62} \text{ Jm}^3$

8. TO FIT AND UNDERSTAND THE MEDIUM AND HEAVY NUCLEAR CHARGE RADII

For atomic number greater than 23, nuclear charge radii [9] can be fitted with the following relation.

$$R_{(Z,A)} \cong \left\{Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right) \tag{30}$$

where $Z \geq 23$ and $\left(G_s m_p / c^2\right) \leq 0.62 \text{ fm}$

See the following table-1.

Table 1: To fit nuclear charge radii.

Proton number	Mass number	Estimated charge radii from relation	Charge radii from reference [9]
26	56	3.718	3.7377
36	86	4.200	4.1835
46	108	4.556	4.5563
56	138	4.900	4.8378
66	148	5.103	5.0455
76	192	5.444	5.4126
86	212	5.653	5.5915

9. NUCLEAR STABILITY AND NUCLEAR BINDING ENERGY AT STABLE MASS NUMBERS

Proton-neutron stability [10] can be understood with the following relation

Let A_S be the stable mass number of Z. If,

$$k \cong \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong \left(\frac{\hbar c}{G_e m_e^2}\right) \cong 1.605 \times 10^{-3},$$

$$\begin{aligned} A_s &\cong 2Z + k(2Z)^2 \cong 2Z + 0.0016(2Z)^2 \\ &\cong 2Z + 0.0064Z^2 \end{aligned} \tag{31}$$

With relation (1) quantitatively it is possible to show that, $(A_s - 2Z) \propto Z^2$ and needs further study at fundamental level. See colum-2 of table-2. With even-odd corrections, accuracy can be improved. Quantitatively this relation can be compared with the computationally proposed relation (8) of reference [10] which takes the following form.

$$N_s \cong 0.968051Z + 0.00658803Z^2 \tag{32}$$

where N_S is the neutron number of a nucleus with atomic number Z on the line of beta stability. Corresponding mass number can be expressed with the following relation.

$$A_s \cong (1 + 0.968051)Z + 0.00658803Z^2 \tag{33}$$

If $Z = 92$, obtained A_S and its actual stable mass number is 238.Considering even-odd corrections, some of the naturally occurring stable atomic nuclides can be fitted with this relation. In some cases there is some discrepancy in fitting the actual stable isotopes. In some cases there is some discrepancy in fitting the actual stable isotopes and we developed alternative procedure. See section-10.

Based on mass number, above relation (31) can also be expressed in the following form.

$$Z \cong \frac{\sqrt{4kA+1}-1}{4k} \tag{34}$$

where A is any mass number. Close to stable atomic nuclides, nuclear binding energy [10] can be understood with the following relation. If $-\left(\frac{3}{5}\left[e^2/4\pi\varepsilon_0 R_p\right]\right) \cong -0.986$ MeV and $-\left(\frac{3}{5}\left[G_s m_p^2/R_p\right]\right) \cong -398.0$ MeV represent the respective self binding energies, then for $(Z \geq 5)$,

$$\begin{aligned} BE &\cong -\left(Z-2+\sqrt{\frac{Z}{30}}\right)\sqrt{\left(\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R_p}\right)\left(\frac{3}{5}\frac{G_s m_p^2}{R_p}\right)} \\ &\cong -\left(Z-2+\sqrt{\frac{Z}{30}}\right)\times 19.8 \text{ MeV} \end{aligned} \tag{35}$$

where R_p is the RMS radius of proton [2]. See the following table-2.

Table 2: Estimated stable mass numbers and their corresponding nuclear binding energy.

Proton number	Estimated Stable mass number	Estimated binding energy in MeV
6	12. 2	88.05
16	33.6	291.6
26	56.3	493.4
40	90.2	775.3
50	116.0	976.0
60	143.0	1176.4
70	171.4	1376.6
82	207.0	1616.7
92	238.2	1816.7
100	264.0	1976.5

10. SEMI EMPIRICAL PROCEDURE FOR ESTIMATING NUCLEAR BINDING ENERGY AND NUCLEAR STABILITY

We would like to stress the fact that, close to stable atomic nuclides starting from Z = 30 to 100, nuclear binding energy is approximately equal to Z times 19.7 MeV. This can be validated from the semi empirical mass formula [10, 11] stability relation.

$$Z \approx \frac{A}{2+\left(a_c/2a_a\right)A^{2/3}} \tag{36}$$

where $\left(a_c/2a_a\right) \cong 0.0157$. With reference to semi empirical mass formula, maximum binding energy per nucleon is close to 8.8 MeV. Based on these two energy constants, there is a scope for understanding nuclear stability. By fitting the data starting from Z = 21 to 92, we try to estimate or predict stable heavy and super heavy atomic nuclides. For example,

1) According to nuclear shell model, Z = 82 seems to be stable at N = 126. Fitted stable mass numbers of Z = 82 are 208 ± 2 and corresponding number of neutrons are 126 ± 2.
2) According to observations, Z = 92 seems to be stable at N = 146. Fitted stable mass numbers of Z = 92 are 238 ± 2 and corresponding number of neutrons are 146 ± 2.
3) According to modern theory, Z = 114 seems to be stable at N = 184 or 196. Predicted stable mass numbers of Z = 114 are 308 ± 2 and corresponding number of neutrons are 194 ± 2. This prediction seems to be in-line with current understanding [12].
4) According to modern theory, Z = 164 seems to be stable at N = 318. Predicted stable mass numbers of Z = 164 are 482 ± 2 and corresponding number of neutrons are 318 ± 2. This prediction also seems to be in-line with current understanding [12].

This semi empirical method involves five important steps as described here. For further details readers can refer our published paper [13].

Step1: To find the approximate binding energy of Z at stability zone. Based on relation (35), for $\left(Z \geq 5\right)$,

$$\left(BE\right)_{A_s} \approx \left(Z+\sqrt{\frac{Z}{30}}-2\right)19.75 \text{ MeV} \tag{37}$$

where $(BE)A_s$ is the approximate nuclear binding energy of Z close to stable mass number, 19.75 MeV can be considered as the characteristic unified nuclear binding energy unit.

Step2: To define a number X in the following way

$$X \approx \frac{(BE)_{A_s}}{8.8 \text{ MeV}} \approx \left(Z + \sqrt{\frac{Z}{30}} - 2\right)\left(\frac{19.75 \text{ MeV}}{8.8 \text{ MeV}}\right) \tag{38}$$

where 8.8 MeV can be considered as the maximum binding energy per nucleon and can be obtained from Iron and Nickel atomic nuclides. With reference to 8.8 MeV, X can be referred to the lowest possible imaginary stable mass number.

Step3: To define a number Y in the following way

$$Y \approx \frac{(X - 2Z)^2}{Z} \tag{39}$$

Step4: To estimate the approximate stable mass number in the following way

$$A_s \approx X + \left(Y^2 + Y\right) \tag{40}$$

where, A_S is the stable mass number of Z.

Step5: To estimate the actual stable mass number range with even – odd corrections

If Z is even and estimated stable mass number A_S is even, then actual stable mass number range can be estimated with the following correction.

$$\text{Actual } A_s \text{ range} \cong \text{Estimated } A_s \pm 2 \tag{41}$$

If Z is even and estimated stable mass number A_S is odd, then actual stable mass number range can be estimated with the following correction

$$\text{Actual } A_s \text{ range} \cong \left(\text{Estimated } A_s - 1\right) \pm 2 \tag{42}$$

If Z is odd and estimated stable mass number A_S is odd, then actual stable mass number range can be estimated with the following correction.

$$\text{Actual } A_s \text{ range} \cong \text{Estimated } A_s \pm 2 \tag{43}$$

If Z is odd and estimated stable mass number A_S is even, then actual stable mass number range can be estimated with the following correction.

$$\text{Actual } A_s \text{ range} \cong \left(\text{Estimated } A_s - 1\right) \pm 2 \tag{44}$$

See the following table-3. Compromising points to be noted are:

1) For even proton numbers, estimated range can be considered as the ‘even-odd stable isotopes’ comprising five number of stable isotopes. For example, if Z = 50, its corresponding estimated stable mass number range is 118 ± 2. It means, stable mass numbers can be (116, 117, 118, 119, 120)
2) For odd proton numbers, estimated range can be considered as ‘odd stable isotopes’ comprising only three isotopes. For example, if Z = 49, its corresponding estimated stable mass number range is 115 ± 2. It means, stable mass numbers can be (113, 115, 117).

Table 3: To fit and estimate medium, heavy and super heavy atomic nuclides.

Proton number	Estimated Binding energy (MeV) at stability zone	Estimated stable mass number with even-odd correction	Actual (stable and long living) isotopes
21	391.8	45 ± 2	45
25	472.3	53 ± 2	55
31	592.8	69 ± 2	69,71
35	673.1	79 ± 2	79,81
41	793.3	93 ± 2	93
47	913.5	109 ± 2	107,109
51	993.5	119 ± 2	121,123
55	1073.5	131 ± 2	133
59	1153.4	141 ± 2	141
60	1173.4	144 ± 2	142,144, 46, 143,145, 148,150
65	1273.3	159 ± 2	159
69	1353.2	169 ± 2	169
75	1473.0	187 ± 2	187,185

81	1592.7	205 ± 2	205,203
86	1692.4	220 ± 2	222
92	1812.1	238 ± 2	238, 235
100	1971.6	262 ± 2	257
106	2091.1	282 ± 2	272
111	2190.7	297 ± 2	283
117	2310.3	317 ± 2	294
118	2330.2	320 ± 2	294
119	2350.1	323 ± 2	
120	2370.0	328 ± 2	

11. MASS AND RADIUS OF A NEUTRON STAR

A) Mass of neutron star

According to G. Srinivasan [14]: "Till this question is resolved all one can say is that the maximum mass of neutron stars is somewhere in the range (1.5 to 6.0) solar masses. It seems to us that the best one can do at present is to appeal to observation".

Let (M_{NS}, m_n) represent masses of neutron star and neutron respectively.

$$\frac{G_N M_{NS} m_n}{\hbar c} \approx \sqrt{\frac{G_s}{G_N}} \tag{45}$$

$$\rightarrow M_{NS} \approx \sqrt{\frac{G_s}{G_N}} \left(\frac{\hbar c}{G_N m_n} \right) \approx 3.17 \text{ Solar mass}$$

Alternatively, it is also noticed that,

$$\frac{G_N M_{NS}^2}{\hbar c} \approx \left(\frac{G_s}{G_N} \right)^2 \tag{46}$$

$$\rightarrow G_N M_{NS} \approx G_s M_{pl}$$
$$\Rightarrow M_{NS} \approx 5.46 \text{ Solar mass}$$

Interesting point to be noted is that,

$$\frac{M_{NS}}{\left(\sqrt{G_N \hbar / c^3}\right)} \approx \frac{G_s}{G_N} \tag{47}$$

From astro-particle physics point of view, it can be given some consideration.

B) Radius of neutron star

It may be noted that, observed masses of neutron stars are of the order of 2 Solar masses and radii are of the order of 11 km [15]. In this context, important point to be noted is that, ratio of neutron star radius and neutron's characteristic radius is of the order of $\sqrt{G_s/G_N}$. It is also possible to say that, ratio of neutron star radius and Planck size is of the order of $\left(G_s/G_N\right)$. It can be expressed in the following way.

$$\frac{R_{NS}}{\left(\sqrt{G_N \hbar / c^3}\right)} \approx \frac{G_s}{G_N} \tag{48}$$

$$\rightarrow R_{NS} \approx \left(\frac{G_s}{G_N}\right)\sqrt{\frac{\hbar G_N}{c^3}} \approx \sqrt{\frac{G_s}{G_N}}\sqrt{\frac{\hbar G_s}{c^3}} \approx 8.1 \text{ km} \tag{49}$$

where $\sqrt{\frac{\hbar G_s}{c^3}} \cong 0.361 \text{ fm}$ can be called as the nuclear Planck length. This can be compared with neutron's positively charged core of radius ~0.3 fm. Now the above relation (49) can be re-expressed in the following way.

$$\frac{\text{Radius of neutron star}}{\text{Nuclear Planck length}} \approx \frac{R_{NS}}{\left(\sqrt{G_s \hbar / c^3}\right)} \approx \sqrt{\frac{G_s}{G_N}} \tag{50}$$

From astro-particle physics point of view, this concept can also be given some consideration.

12. 'SYSTEM OF UNITS' INDEPENDENT AVOGADRO NUMBER AND MOLAR MASS UNIT

If, atoms as a whole believed to exhibit electromagnetic interaction, then molar mass constant and Avogadro number, both can be understood with the following simple relation.

$$G_e\left(m_{atom}\right)^2 \cong G_N\left(M_{mole}\right)^2 \tag{51}$$

where m_{atom} is the unified atomic mass unit and M_{mole} is the molar mass unit or gram mole.

Thus it is very clear to say that, directly and indirectly 'gravity' plays a key role in understanding the molar mass unit.

$$\frac{M_{mole}}{m_{atom}} \cong \sqrt{\frac{G_e}{G_N}} \rightarrow M_{mole} \cong \sqrt{\frac{G_e}{G_N}} \times m_{atom} \tag{52}$$

where $\sqrt{\frac{G_e}{G_N}} \cong 5.96\times10^{23}$ and $\left(0.00099 > M_{mole} < 0.001\right)$ kg

Based on these relations, "independent of system of units" and "independent of ad-hoc selection of exactly one gram", it may be possible to explore the correct physical meaning of the famous 'Molar mass unit' and 'Avogadro number' in a unified approach [7].

13. TO FIT AND UNDERSTAND THE ATOMIC RADII

Considering the geometric mean of the two assumed gravitational constants associated with proton and 'atom as whole', atomic radii can be fitted in the following way. By following the periodic arrangement of atoms and their electronic arrangement, accuracy can be improved.

$$\begin{aligned} R_{atom} &\cong A_s^{1/3}\sqrt{\left(\frac{2G_s m_n}{c^2}\right)\left(\frac{2G_e m_{atom}}{c^2}\right)} \\ &\cong A_s^{1/3} * 33.0 \text{ pico.meter} \end{aligned} \tag{53}$$

where A_S is the stable atomic mass number of the atom, m_n is the average mass of nucleon and m_{atom} is the unified atomic mass unit. Note that, this relation resembles the famous relation for nuclear radii proposed by Rutherford [16, 17]. See the following table-4.

Table 4: Estimated atomic radii.

Proton number	Stable Mass number	Estimated atomic radii (pico meter)	Reference data [18] (pico meter)
1	1	33.0	31
6	12	75.6	76
16	32	104.8	105
27	57	127.0	126
28	62	130.6	124
29	63	131.3	132
30	66	133.4	122
40	90	147.9	175
47	107	156.7	145
60	142	172.2	201
70	172	183.5	187
81	203	193.9	145
89	227	201.3	215
92	238	204.5	196

14. TO FIT AND UNDERSTAND THE BOHR RADIUS

With reference to relation (23), $\left(\frac{R_0}{2}\right) \cong \left(\frac{G_s m_p}{c^2}\right)$ and by considering $\left(\frac{1}{2n^2}\right)$ as the probability of finding electron in its orbits labelled as $n = 1,2,3,.,$ total energy of electron can be understood with the following relation.

$$\begin{aligned}\left(E_{tot}\right)_n &\cong -\left(\frac{1}{2n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0\left(G_s m_p / c^2\right)}\right)\\ &\cong -\left(\frac{1}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0\left(2G_s m_p / c^2\right)}\right)\\ &\quad -\left(\frac{1}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right)\end{aligned} \tag{54}$$

Here interesting point to be noted is that, $\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right) \cong 1.171\times 10^{-5}$ can be considered as the electromagnetic and gravitational force ratio pertaining to electron where the operating gravitational constant is G_e

Discrete Bohr radii of electron can be understood with:

$$(a_0)_n \cong n^2\left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}\right)\left(\frac{G_s m_p}{c^2}\right) \tag{55}$$

Bohr radius can be understood with:

$$\begin{aligned} a_0 &\cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}\right)\left(\frac{G_s m_p}{c^2}\right) \\ &\cong \left(\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}\right)\left(\frac{R_0}{2}\right) \end{aligned} \tag{56}$$

16. DISCUSSION AND CONCLUSION

By introducing two pseudo gravitational constants, we make an attempt to combine the old ‘strong gravity’ concept with ‘Newtonian gravity’ and try to understand and re-interpret the constructional features of nuclei, atoms, and neutron stars in a unified approach and finally making an attempt to estimate the Newtonian gravitational constant from the known elementary atomic and nuclear physical constants.

In an advanced and in a semi empirical approach, we proposed peculiar relations (1) to (56). Considering the wide applicable range of the proposed two assumptions, we are confident to say that, with further research and analysis, ‘hidden and left over physics’ can easily be explored. In this context, we would also like to stress the fact that, with current understanding of String theory [19] or Quantum gravity [20], qualitatively or quantitatively, one cannot implement the Newtonian gravitational constant in microscopic physics. This ‘drawback’ can be considered as a characteristic ‘inadequacy’ of modern unification paradigm. Proceeding further, with reference to String theory models and Quantum gravity models, proposed two pseudo gravitational constants and presented semi empirical relations can be given some consideration in developing a ‘workable model’ of ‘final unification’.

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